The Setting of Macroprudential Instruments in the Eurozone: An Estimated DSGE Analysis

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Abstract

How macroprudential instrument should be set in a federal structure such as the Eurozone? We estimate a two-country DSGE model accounting for alternative macroprudential instruments and implementations schemes. Our main conclusions underline a possible conflict between the federal and the national levels in the implementation of heterogenous macroprudential measures based on a single instrument as the Pareto optimal situation computed on a federal ground requires an asymmetric choice of instruments, while more symmetric practices should be preferred on regional ground. The adoption of combined instruments in each country solves this potential conflict as it leads to both a higher welfare increase in the Pareto optimal equilibrium and always incurs national welfare gains. However, in all cases, the Pareto optimal equilibrium cannot be reached on national incentives and requires a supranational enforcing mechanism.

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1 Introduction

The recent financial crisis has led Eurozone countries to reform their supervisory framework. New institutions have been established following the report of De Larosière (2009). The new scheme is organized on two pillars: the first pillar is devoted to macro-prudential supervision comprising the European Systemic Risk Board (ESRB) and a second pillar devoted to micro-prudential supervision, comprising three different European Supervisory Authorities (ESAs) – one for banking, one for insurance and one for the securities markets.

The main objective of the ESRB is to promote a coordinated implementation of macroprudential measures among the members of the Eurozone so as to provide a more robust macro-prudential supervisory framework. The organization of the European macroprudential scheme is original. First, it is the only supranational setup yet to be observed. Owing to their mutual financial integration, European countries have taken into account the fact that financial stability should be treated as a public good and that isolated national actions may undermine the conduct of an appropriate macroprudential policy. Second, it is the only macroeconomic policy in the Eurozone that accounts for country heterogeneities, as monetary policy reacts to Union wide aggregates while the Stability and Growth Pact (SGP) imposes common rules on national public debts and deficits.

The aim of this paper is to evaluate how macroprudential instruments should be set in a monetary union such as the Eurozone, given this federal organization. We more particularly assess the degree of symmetry that should be promoted in the conduct of macroprudential policies. We develop a two-country DSGE model that accounts for some major features of the Eurozone regarding the problem at hands (key role of the banking system, cross-border bank loans, heterogeneity credit and business cycles) to provide a quantitative evaluation of welfare gains coming from the implementation of macroprudential measures. In this model, heterogeneity between the member countries of the Eurozone is accounted for by distinguishing core countries and peripheral countries.

1For a synthetic presentation of the various institutional situations observed in real life practices regarding the definition of the macroprudential mandate, see (Nier et al., 2011). In some cases, it involves a reconsideration of the institutional boundaries between central banks and financial regulatory agencies (or the creation of dedicated policymaking committees), while in other cases, efforts are made to favor the cooperation of authorities within the existing institutional structure. (Nier et al., 2011) find that the vast majority of arrangements that are in place or are being developed across countries can be organized in seven models, which in turn form three broad groups of models that differ in the degree of institutional integration between central bank and regulatory agencies.

2The criterion to divide the Eurozone in two blocks is discussed in the following section. Core countries: Austria, Belgium, Germany, Finland, France, Luxembourg and Netherlands. Peripheral countries: Spain, Greece, Ireland, Italy and Portugal.
We introduce three major features of the Eurozone in the analysis. First, we account for the key role of cross border loans in creating spillovers that lead to the diffusion of national macroprudential measures to other countries. In line with the data, we distinguish between interbank and corporate cross border loans. Second, we assume that, according to the indicative list of macroprudential instruments issued by the ESRB in 2013, national macroprudential authorities can use alternative instruments to affect either the lending or borrowing conditions of their country. We more particularly contrast the use of single vs multiple instruments in the conduct of macroprudential measures. Third, we consider a granular implementation of macroprudential policies designed to address the development of national financial imbalances.

The model, estimated with Bayesian methods on Eurozone quarterly data over the sample period 1999Q1 to 2013Q3, leads to the following results: First we report a possible conflict between the federal and the national levels in the implementation of heterogenous macroprudential measures based on a single instrument. On federal ground, the Pareto optimal situation requires an asymmetric choice of instruments. However, this is a no free lunch situation as it may create regional welfare losses. On regional grounds, more symmetric practices should be preferred to provide welfare gains to all the participating countries. Second, The adoption of combined instruments in each country solves this potential conflict as it leads to both a higher welfare increase in the Pareto optimal equilibrium and always incurs national welfare gains. Third, the Pareto optimal equilibrium cannot be reached on national incentives, Thus a supranational enforcing mechanism such as the one introduced by the ESRB is necessary independently of the nature and number of macroprudential instruments adopted in each country.

The paper is organized as follows: Section 2 describes the institutional background and some stylized facts regarding the situation of the Eurozone. Section 3 outlines the model. Section 4 presents the econometric results. Section 5 describes macroprudential instruments. Section 6 presents the welfare and macroeconomic consequences of alternative macroprudential regimes. Section 7 evaluates the sensitivity of the results with respect to cross border lending. Finally section 8 concludes.

2 The Institutional Background

This section sketches the main institutional aspects regarding the conduct of macroprudential policy in the Eurozone and briefly discusses the interest of adopting heterogeneous/homogenous macroprudential measures between participating countries.

2.1 The Federal Organization of Macroprudential Policy

At the federal level, the recent financial crisis revealed serious shortcomings in the conduct of financial supervision in the Eurozone as the growing integration of European financial markets and the increase in cross border banking that followed the adoption of the Euro has not been met by the adoption of similar legislations in the participating countries. In particular the implementation and enforcement of this legislation has ultimately been left to the discretion of Member States supervisors, according to the principle of home country control and mutual recognition.

The European Systemic Risk Board (ESRB) is aimed at providing tools to coordinate national macroprudential policies. The ESRB cannot use macro-prudential instruments directly nor it has binding powers to impose the policy to nations belonging to this organization. Instead, it can issue warnings and recommendations to national authorities and to EU institutions. Through the warnings or recommendations, the macro-prudential concerns of the ESRB should be transformed into action by other authorities or bodies. However, the recommendations are supported by an ‘act or explain’ mechanism where the addressees are obliged to provide a justification in case they do not follow the recommendations. If the ESRB considers that the reaction is inadequate, it should inform, subject to strict confidentiality rules, the addressees, the Council and, where appropriate, the European Supervisory Authority concerned.

As underlined by the ESRB (commentary 2) macro-prudential policy also has an important national component. First, because systemic risks can arise at the national (or sectoral) level, as financial cycles and the structural characteristics of financial systems typically differ between countries, and thus may require a different policy response. Second, because the responsibility for the adoption of

\[4\text{The ESRB has two instruments to carry out its mandate, namely warnings and recommendations. The difference between them is that warnings call for the attention of the addressees to identified systemic risks, without a detailed description of the actions required, whereas recommendations include advice on policy actions to be taken to mitigate the identified risks. Addressees of the ESRB's warnings and recommendations can be the European Union, individual EU Member States and the three ESAs, as well as national supervisory authorities in the EU. Recommendations may also be addressed to the European Commission in respect of the relevant EU legislation. Warnings and recommendations can either be confidential, and thus communicated only to the targeted addressees, or they can be public.}\]
the measures necessary to maintain financial stability lies within national frameworks. Therefore, the effectiveness of macro-prudential policy in Europe depends not only on the institutional structure at the EU level, but also on the institutional frameworks and policy mandates at the level of individual Member States.

The third ESRB recommendation deals with the macro-prudential mandate for national authorities and provides a set of guiding principles for the macro-prudential frameworks in the EU Member States. This authority should have sufficient powers to pursue macro-prudential policy and the necessary independence to fulfill its tasks. Recently the ESRB has proposed a classification of policy instruments according to their specific targets as well as possible selection of a more limited set of core instruments. (since April 2013, the ESRB has provided recommendations on an indicative list of macroprudential instruments)

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2.2 The granular implementation of macroprudential measures

The granular implementation of macroprudential measures in the Eurozone is original with respect to the conduct monetary and fiscal policies. In the later two cases, countries are treated uniformly. Monetary policy reacts to union wide aggregates, while national fiscal policy should respect ratios regarding deficit and debts uniformly set for all countries participating to the Eurozone.

As an original founding principle of the ESRB, macroprudential policy is tailored to the situation of countries to fit the heterogeneity of national financial cycles. This heterogeneity is underlined in Figure 1 by contrasting Eurozone core and peripheral countries according to their status in terms of surplus or deficit of their current account. As reported main differences characterize financial developments in these two groups of countries. In particular, peripheral countries experienced an explosive growth of credit followed by a sharp drop as well as a rise of the corporate bond yield. These heterogenous financial developments between core and peripheral countries may, in turn, require alternative macroprudential measures.

The problem faced by the ESRB is to accommodate these heterogeneous national financial development with more homogenous national practices than encountered before the recent financial crisis. As shown in Figure 2, different policy initiatives have been taken since the creation of the ESRB, and a road map has been set to enhance more homogenous practices among countries participating

\[ \text{5} \]In practice, Countries can set particular values for the different instruments. The rule is at follows: the national macro-prudential authorities in the EU should be able to tighten settings of instruments to levels above those provided for in EU legislation in a timely manner based on local conditions.
Figure 1: Regional divergences in the UEM

to this structure. As underlined by ESRB (2013, report October), this calendar can be analyzed as providing a smooth transition towards a more centralized and symmetric system.

The adoption of more homogenous practices is a debated question. On one hand the interest of keeping heterogeneous macroprudential policy measures allows a granular treatment of financial risk in the Eurozone. As macro-prudential policies can be targeted at specific sectors or regional developments, they can help attenuate the credit cycle heterogeneity that characterizes the euro area and support a more balanced diffusion of monetary policy developments in the Eurozone.

On the other hand, the move towards more symmetric practices in the conduct of macroprudential practices can be justified on an economic ground as countries belonging to the Eurozone form an integrated financial area. In this area, banks provide the main liquidity to the system and cross-border banking has reached a value representing 24% of Eurozone GDP before the financial crisis of 2008. The transfer of macroprudential powers to the Federal level can thus make sense as cross-border banking activities have increased the interconnection of financial decisions in the Eurozone. Setting more homogenous macroprudential rules at the union level can be considered as a solution to a problem of externalities. Externalities arise from the fact that national authorities do not internalize their contribution to federal financial instability. Finally, the Single Supervisory Mechanism (SSM) initiative that promotes a uniform regulation of the banking system across the Eurozone may modify the original organization of macroprudential implementation for countries participating to this structure. As noted by shoemaker (2013) even if a centralized model should not imply a uniform application of the
The calendar

December 2011: The ESRB recommends Member States to establish a legal mandate for national macro-prudential authorities by June 2013.

December 2012: The Council agreed on a regulation creating a single supervisory mechanism (SSM) which will be responsible for the micro and macro-prudential supervision of all banks in the participating countries with the ECB acting as European supervisor “responsible for the effective and consistent functioning” of the mechanism.

April 2013: The ESRB recommends Member States to ensure a minimum set of instruments is available, and identifies a common benchmark for intermediate objectives and instruments.


Summer 2014: The SSM acquires some macro-prudential powers for instruments included in the CRD4/CRR, namely those related to banks.

December 2014: Deadline ESRB recommendation on intermediate objectives and instruments.

December 2015: Deadline ESRB recommendation on strategy.

Figure 2: Road map of the newly created institutions in the European Union

macroprudential tools across the countries in the SSM, in some instances, the European Central Bank may wish to apply a uniform macro-prudential requirement when a particular asset is increasing too fast in many SSM countries.

3 The Analytical Framework

This section introduces a two-country DSGE model that accounts for the main specificities of the Eurozone for the question at hands (namely financial heterogeneities and cross-border loans). Our framework describes a monetary union made of two asymmetric areas \( i \in \{c, p\} \) (where \( c \) is for core and \( p \) for periphery parts) of relative sizes \( n_c \) and \( n_p \). As shown in Figure 4, each part of the monetary union is populated by consumers, intermediate and final producers, entrepreneurs, capital suppliers and a banking system. Regarding the conduct of macroeconomic policy, we assume national fiscal authorities and a common central bank. We present the model anticipating the symmetric equilibrium across households, firms and banks that populate the economy.

\( ^6 \) The whole model is presented in appendix.
Households The representative household supplies $H_{i,t}$ hours of work, saves $D^d_{i,t}$ and maximizes utility intertemporally $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t e^{\epsilon^{i}_{t,t+\tau}} \mathcal{U}(C_{i,t+\tau}, H_{i,t+\tau})$, where $C_{i,t}$ is the consumption, $\beta \in (0, 1)$ is the subjective discount factor and $\epsilon^{i}_{t,t}$ is an exogenous shock to preferences. The period utility function takes the form $\mathcal{U}(C_{i,t}, H_{i,t}) \equiv (C_{i,t} - h^C_{i,t} C_{i,t-1})^{1-\sigma^C} / (1 - \sigma^C) - \chi^C_i H_{i,t}^{1+\sigma^L} / (1 + \sigma^L)$ where $\sigma^L \geq 0$ is the curvature coefficient in the disutility of labor, $\sigma^C \geq 0$ is the risk aversion coefficient and $h^C_i \in [0, 1]$ are external consumption habits. The consumption basket of the representative household is composed of home and foreign goods $C_{i,t} = ((1 - \alpha^C_i)^{1/\mu} C_{hi,t}^{(\mu-1)/\mu} + (\alpha^C_i)^{1/\mu} C_{fi,t}^{(\mu-1)/\mu})^{\mu/\mu} - 1)$ where $1 - \alpha^C_i > 1/2$ is the home bias in consumption and $\mu \geq 0$ is the elasticity of substitution between home and foreign goods.

Firms There is a continuum of monopolistically competitive firms, each producing differentiated goods using hours of work and capital inputs $K_{i,t}$ and set production prices $P_{i,t}$ according to the Calvo model. Output supplied by firms is $Y_{i,t} = \varepsilon^{A}_{i,t} K_{i,t}^{\alpha} H_{i,t}^{1-\alpha}$ where $\varepsilon^{A}_{i,t}$ is an innovation to the productivity and $\alpha \in [0, 1]$ is the share of capital services in the production. According to the Calvo mechanism, each period firms are not allowed to reoptimize the selling price with probability $\theta^p_i \in [0, 1)$ but price increases of $\xi^p_i \in [0, 1)$ at last period’s rate of price inflation, $P_{i,t} = \pi_{i,t-1} P_{i,t-1}$ where $\pi_{i,t} = P_{i,t}/P_{i,t-1}$. Under this setting, it is possible to derive the aggregate inflation rate of production goods, it is defined by the function, $\pi_{i,t} = \int (\mathbb{E}_t \pi_{i,t+1}, \pi_{i,t-1}, mc_{i,t})$ where $mc_{i,t}$ is the marginal cost of production.

Entrepreneurs We add a financial constraint to the producing firms in order to implement a banking sector in the model as Poutineau and Vermandel (2013). We standardly introduce an entrepreneurial sector that buys capital at price $Q_t$ in $t$ and uses that capital in the production in period $t + 1$. Under this assumption, the capital arbitrage equation implies that the expected rate of return on capital is given by $1 + R^{L}_{i,t+1} = \mathbb{E}_t \left[ Z_{i,t+1} + (1 - \delta) Q_{i,t+1} \right] / Q_{i,t} \gamma$ where $\delta \in [0, 1]$ and and $Z_{i,t}$ is respectively the depreciation rate and the marginal product of capital. Assuming that entrepreneurs are credit-constrained, they finance capital by their net wealth $N_{i,t}$ and lending $L^{H}_{i,t+1}$ subject to external habits $h^L_i \in [0, 1)$. The balance sheet of the entrepreneur then writes, $Q_{i,t} K_{i,t+1} = L^{H}_{i,t+1} + N_{i,t+1}$. The entrepreneur has access to domestic and foreign banks to meet its balance sheet, $L_{i,t+1} = ((1 - \alpha^L_i)^{1/\nu} L_{hi,t+1}^{(\nu-1)/\nu} + (\alpha^L_i)^{1/\nu} L_{fi,t+1}^{(\nu-1)/\nu})^{\nu/\nu}$ where $\alpha^L_i$ represents the percentage of cross-border loan flows in the monetary union. The total cost of loans, $P_{i,t} L_{i,t}$ is thus defined according to, $\left(1 + P^{L}_{i,t} \right)^{1-\nu} = \left(1 - \alpha^L_i \right) \left(1 + R^{L}_{h,i,t} \right)^{1-\nu} + \alpha^L_i \left(1 + R^{L}_{f,i} \right)^{1-\nu}$ where $R^{L}_{h,i}$ denotes the credit rate set by bank in country $i$.

The representative entrepreneur conducts a mass $\omega \in [\omega_{\text{min}}, +\infty)$ of heteroge-
nous investment projects drawn from Pareto distribution. The rentability of the \( \omega^{\text{th}} \) investment project is, \( \omega \left( 1 + R^{\text{th}}_{i,t+1} \right) \). There is a critical project \( \omega_{i,t}^{C} \) that determines the threshold of profitability of the firm. Aggregating projects above the threshold, we can compute the share of profitable projects \( n_{i,t}^{E} \) in the economy \( i \). Supposing that entrepreneurs are optimistic as De Grauwe (2010) regarding their expected aggregated return \( \omega \) on investment projects, we find the financial acceleration equation as in Bernanke et al. (1999). The external finance premium drives a wedge between the expected return on capital and the expected return demanded by banks and takes the form, \( R^{E}_{i,t} - P^{L}_{i,t} \approx \kappa_{i} f \left( Q_{i,t} K_{i,t+1} / N_{i,t+1} \right) \) where \( \kappa_{i} > 0 \) measures the elasticity of the premium with respect to leverage.

**Banks** In each country, the banking sector finances investment projects to home and foreign entrepreneurs by supplying one-period loans. As in Poutineau and Vermandel (2015), the banking system is heterogenous with regard to liquidity, and banks engage in interbank lending at the national and international levels. To introduce an interbank market, we suppose that the banking system combines liquid and illiquid banks. Thus, cross-border loans are made of corporate loans (between banks and entrepreneurs) and interbank loans (between liquid and illiquid banks). We assume that banks distributed over \([0, \lambda] \) are illiquid (i.e. credit constrained), while the remaining banks distributed over share \([\lambda, n_{i}] \) are liquid and supply loans to entrepreneurs and to illiquid banks. We assume that a liquid bank is characterized by her direct accessibility to the ECB fundings. Conversely, an illiquid bank does not have access to the ECB fundings. According to this assumption, the balance sheet of the illiquid banks is \( L_{i,t+1}^{L} = D_{i,t} + IB_{i,t+1}^{H} + BK_{i,t+1}^{L} + E_{i,t} \) while the balance sheet of the liquid bank is \( L_{i,t+1}^{L} + IB_{i,t+1}^{H} = D_{i,t} + L_{i,t+1}^{ECB} + BK_{i,t+1}^{L} + liab_{i,t} \), where \( L_{i,t+1}^{L} \) is the loan supply of borrowing banks, \( D_{i,t} \) is the amount of households deposits, \( IB_{i,t+1}^{H} \) is the interbank loans supplied by liquid banks subject to external habits at a degree \( h_{i,t}^{B} \in [0, 1] \), \( IB_{i,t+1}^{L} \) is the supply of interbank loans, \( BK_{i,t+1}^{L} \) is the bank capital, \( E_{i,t} \) are other liabilities in the balance sheet of the bank and \( L_{i,t+1}^{ECB} \) is the amount of refinancing loans supplied by the central bank. The total amount borrowed by the representative bank writes, \( IB_{i,t+1}^{L} = \left( 1 - \alpha_{i}^{LB} \right)^{1/\xi} IB_{i,t+1}^{H} + \left( \alpha_{i}^{LB} \right)^{1/\xi} IB_{i,t+1}^{L} \), where parameter \( \xi \geq 0 \) is the elasticity of substitution between domestic and foreign interbank funds, \( \alpha_{i}^{LB} \in [0, 1] \) represents the percentage of cross-border interbank loan flows in the monetary union. The total cost incurred by illiquid banks to finance interbank loans, \( P^{IB}_{i,t} \), is thus defined according to the CES aggregator, \( \left( 1 + P^{IB}_{i,t} \right)^{1-\xi} = \left( 1 - \alpha_{i}^{LB} \right) \left( 1 + R^{h}_{i,t} \right)^{1-\xi} + \alpha_{i}^{LB} \left( 1 + R^{IB}_{i,t} \right)^{1-\xi} \), where \( R^{LB}_{i,t} \) (resp. \( R^{IB}_{i,t} \)) is the cost of loans obtained from home (resp. foreign) banks in country \( i \). Following Hirakata et al. (2009), the bank capital accumulation process of illiquid banks \( BK_{i,t+1}^{L} \) is determined by, \( BK_{i,t+1}^{L} = \left( 1 - \tau^{BK} \right) \Pi_{i,t}^{B} \), where \( \tau^{BK} \) is a pro-
portional tax on the profits of the bank. Concerning macroprudential policy, we suppose that banks pay a capital requirement cost $F_i(\cdot)$ when their bank capital-to-assets ratio deviates from its steady state value. Regarding the interbank rate, the representative liquid bank decides the interbank rate by using a convex monitoring technology à la Calvo and Woodford (2010) denoted $AC_{i,t+1}^{DB}(b) = f(IB_{i,t+1}^s)$, in equilibrium the interbank rate is $R_{i,t}^{IB} = R_t + f'(IB_{i,t+1}^s)$. In the same vein as Darracq-Pariès et al. (2011), we measure the pass-through of interest rates by supposing that the representative bank sets the deposit and credit rates in staggered basis à la Calvo. Letting $\theta^L_i$ (\theta^D_i$) denotes the country specific probability of the bank not being able to reset its deposit (credit) interest rate. The aggregate deposit rate writes, $R_{i,t}^{D} = f(\mathbb{E}_t R_{i,t+1}^{D}, R_t, \varepsilon_{i,t}^D)$ where $\varepsilon_{i,t}^D$ is a ad hoc mark-up shock and $R_t$ is the (tailed) ECB refinancing rate. The New Keynesian Phillips Curve for deposit rates implies that the expected future rate depends on the refinancing rate markup and exogenous shock. Similarly, the aggregate credit rate is defined by, $R_{i,t}^{I} = f(\mathbb{E}_t R_{i,t+1}^{I}, R_t, P_{i,t}^{IB}, F_{i,t}^{IB}, \varepsilon_{i,t}^{IB})$. Solving forward $R_{i,t}^{I}$, one can see that current and expected future ECB rate $R_t$ and firms default profitability $\mathbb{E}_t \eta_{i,t+1}$, interbank rate $P_{i,t}^{IB}$, Basel I capital requirement costs $F_{i,t}^{IB}$ and an ad hoc mark-up shock $\varepsilon_{i,t}^{IB}$ drive today’s credit rates.

**Capital Suppliers** The representative capital producer buys depreciated capital stock $(1-\delta)K_{i,t}$ and investment goods $I_{i,t}$ and produces new capital goods $K_{i,t+1}$ at a price $Q_{i,t}$. Capital supplier buys home and foreign investment goods, $I_{i,t} = \left( (1-\alpha^I_t) \right)^{\mu} I^{(\mu-1)/\mu}_{i,t} + (\alpha^I_t) \left( \frac{\mu}{\mu} \right) I^{(\mu-1)/\mu}_{i,t}$ where $1-\alpha^I_t > 0.5$ is the home bias in its consumption basket.

**Monetary Policy** Finally, monetary authorities choose the nominal interest rate according to a standard Taylor rule $R_t = f(\pi^C_{u,t}, \Delta Y_{u,t}, \varepsilon_t^R)$ where $\pi^C_{u,t}$ and $\Delta Y_{u,t}$ are the growth rates of price and GDP of the monetary union and $\varepsilon_t^R$ is an exogenous monetary policy shock.

**Shocks and Equilibrium Conditions** In this model, there are 10 country specific structural shocks for each area $s = \{A, G, U, P, W, I, N, L, D, B\}$ and one common shock in the Taylor rule. The shocks follow a first order autoregressive process such that $\varepsilon^s_{i,t} = \rho^s_{i} \varepsilon^s_{i,t-1} + \eta^s_{i,t}$ and $\varepsilon_t^R = \rho^R \varepsilon_{t-1}^R + \eta_t^R$. In these first-order autoregressive processes, $\rho^s_{i}$ and $\rho^R$ are autoregressive roots of the exogenous variables. Standard errors $\eta^s_{i,t}$ and $\eta_t^R$ are mutually independent, serially uncorrelated and normally distributed with zero mean and variances $\sigma^2_{s,t}$ and $\sigma^2_R$ respectively. A general equilibrium is defined as a sequence of quantities $\{Q_t\}_{t=0}^{\infty}$ and prices $\{P_t\}_{t=0}^{\infty}$ such that for a given sequence of quantities $\{Q_t\}_{t=0}^{\infty}$ and the realization
of shocks \( \{ S_t \}_{t=0}^{\infty} \), the sequence \( \{ P_t \}_{t=0}^{\infty} \) guarantees the equilibrium on the capital, labor, loan, intermediate goods and final goods markets.

After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) substituting the relevant demand functions, (iv) normalizing the total size of the monetary union \((n_c + n_p = 1)\) such that the size of the core area is \(n\) and the peripheral area size is \(1 - n\), we can deduct the general equilibrium conditions of the model detailed in appendix.

4 Estimation and Empirical Performance

4.1 Data

The model is estimated with Bayesian methods on Euro Area quarterly data over the sample period 1999Q1 to 2013Q3. The dataset includes 17 time series: real GDP, real consumption, real investment, the ECB refinancing operation rate, the consumption deflator, the real unit labor cost index, the real index of notional stocks of corporate and interbank loans, and the real borrowing cost of non-financial corporations. Data with a trend are made stationary using a linear trend and are divided by the population. We also demean the data because we do not use the information contained in the observable mean. Figure 5 plots the transformed data.

4.2 Calibration and Priors

The complete set of calibrated parameters is reported in Table 1. We fix a small number of parameters commonly used in the literature of real business cycles models\(^7\): these include \(\beta\) the discount factor, \(\delta\) the quarterly depreciation rate, \(\alpha\) the capital share in the production and the share of steady state hours worked \(\bar{H}_i\). The government expenditures to GDP ratio is set at 24%\(^8\). Concerning the substitutability of good and wage varieties, it is calibrated as in Smets and Wouters (2007) at 10 which roughly implies a markup of 11%. Regarding financial parameters, we fix \(\bar{N}/\bar{K}\) the net worth to capital ratio of entrepreneur near Gerali et al. (2010). The steady state value of spreads \((\bar{R} - \bar{R}^D\) and \(\bar{R}^L - \bar{R}^D)\) and the bank balance sheet \((\bar{D}/\bar{L}, \bar{BK}/\bar{A}\) and \(\bar{TB}_d/\bar{A})\) are calibrated on their average values observed in the data. The steady state interbank rate is assumed to be

\(^7\)The Euro area was created in 1999, so our sample is relatively short, following Smets and Wouters (2007), we calibrate rather than estimate structural parameters which are known to be weakly identified (we do not estimate parameters that determine the steady state of the model).

\(^8\)On average, Euro Area households consumption represents 56% of the GDP and investment 20%, then the exogenous spending-GDP ratio is 24%. 
Table 1: Calibration of the model (all parameters are on a quarterly basis). Note: \( \hat{A} \) term denotes the assets of the bank, it is \( \hat{L}^s \) for illiquid banks and \( IB^s + \hat{L}^s \) for liquid ones.

Equal to the ECB refinancing rate \( R = R^{IB} = R^{IB}_p \). The annual share of insolvent entrepreneurs’ projects \( \bar{\eta}^E \) is fixed at 2.5% and the quarterly cost of audit \( \mu^b \) is 0.10, those values are comparable to Bernanke et al. (1999) and Hirakata et al. (2009). Following Kolasa (2009), we set the parameter governing the relative size of the core area \( n_c \) to 58%, which is the share implied by nominal GDP levels averaged over the period 1999-2013. The portfolio cost calibration \( \chi^D \) is based on the findings of Schmitt-Grohé and Uribe (2003) and is deemed necessary to close open economy models. Finally, we find that parameters driving the substitutability between home and foreign loans are weakly identified due to their small impacts on the likelihood, thus we assume that loans are slightly substitutable such that \( \nu, \zeta = 1.10 \).

Our priors are listed in Table 8. Overall, they are either relatively uninformative or consistent with earlier contributions to Bayesian estimations. For a majority of new Keynesian models’ parameters, i.e. \( \sigma^C, \sigma^L, h_i^C, \theta_i^P, \xi_i^P, \theta_i^W, \xi_i^W, \psi_i, \chi_i^L, \phi^p, \phi^\Delta \) and shocks processes parameters, we use the prior distributions close to Smets and Wouters (2003, 2007) and Kolasa (2009). All shocks process parameters are
taken from Smets and Wouters (2007) except for the productivity shocks, we use priors similar to Smets and Wouters (2003). The Calvo probabilities are assumed to be around 0.75 for prices and wages as in Smets and Wouters (2003), while credit rates and deposit rates priors rely largely on Darracq-Pariès et al. (2011) with a mean of 0.50 and a standard deviation of 0.10. Concerning international macro-economic parameters, our priors are inspired by Lubik and Schorfheide (2006). For the final goods market openness $\alpha^C_i$ and $\alpha^I_i$, we choose priors in line with the findings of Eyquem and Poutineau (2010). The substitutability between different goods varieties is set at 1.50 with for standard deviation 0.50 which is inspired by Lubik and Schorfheide (2006). For the credit market openness, we choose priors coherent with the observed market openness over the period 1999-2013 and differ between Core and Periphery. The prior distribution mean of the corporate loan market openness is set at 4% the core area and 8% for the Periphery. In the same vein, the interbank market openness is 20% and 25% for the Core and Periphery respectively. We set the prior for the elasticity of the external finance premium $\kappa_i$ to a beta distribution with prior mean equal to 0.05 and standard deviation 0.03 consistent with previous financial accelerator estimations (Gilchrist et al., 2009a; Bailliu et al., 2012). In order to catch up the correlation and co-movement between countries’ aggregates, we estimate the cross-country correlation between structural shocks. Our priors are inspired by in Jondeau et al. (2006) and Kolasa (2009), we set the mean of the prior distribution for the shock correlations between core countries and peripheral countries to 0.20 with a standard deviation of 0.20. For loan demand habits for firms and banks, we chose a very uninformative prior of mean 0.50 and standard deviation 0.15 with a beta distribution. We use a similar uninformative prior for the share of illiquid bank. The capital requirement cost $\chi^K_i$ has a prior mean of 15 with standard deviation 2.50 which is between the assumption of Gerali et al. (2010) and Darracq-Pariès et al. (2011). Finally, the monitoring cost $\chi^B_i$ on the interbank market is set to a normal distribution with mean 0.05 and variance 0.02 with an inverse gamma distribution, which is consistent with Cúrdia and Woodford (2010).

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9This prior on the productivity is necessary in the fit exercise to obtain a converging MCMC algorithm.
4.3 Posterior and Fit of the model

The methodology is standard to the Bayesian estimations of DSGE models\(^{10}\). Table 8 reports the prior and posterior distributions of the parameters of the model. Overall, all estimated structural parameters are significantly different from zero. Comparing our estimates of deep parameters with the baseline of Smets and Wouters (2003) for the Euro Area, we find higher standard deviations for most of the shocks, this mainly comes from the 2007 financial crisis captured by our model as strong demand shocks followed by persistent financial shocks. Concerning the parameters characterizing the investment adjustment cost, Calvo wage, consumption habits, labour disutility and the weight on output growth, our estimates are also very close to Smets and Wouters (2003). Turning to the degree of price stickiness, the monetary policy smoothing and the weight on inflation, our posterior distributions are close to the estimates of Christiano et al. (2010).

The main differences between core and peripheral countries explain the divergence of business cycles since the Eurozone creation. The gap between core and periphery originates from both shocks and structural parameters. Estimated standard deviation of real shocks are similar between the two areas, while nominal and financial shocks is larger in peripheral countries. The persistence of shocks is similar between countries except for the preference shock: households in peripheral countries experience large and volatile innovations. The disynchronization of the business cycles are also driven by the capital utilization cost elasticity, the external finance premium elasticity and the demand habits on the financial markets. The diffusion of monetary policy is not homogenous and symmetric, particularly for the deposit market where the rate stickiness is more important in the peripheral area than in the core one.

Concerning the home bias in the consumption, investment, corporate loans and interbank loans baskets, the model’s estimates are consistent with the data, sug-

\(^{10}\)Interest rates data are annualized, we take into account this maturity by multiplying by 4 the rates in the measurement equation. The number of shocks and observable variables are the same to avoid stochastic singularity issue. Recalling that \(i \in \{h, f\}\), the vectors of observables \(Y_{\text{obs}} = [\Delta \log \hat{Y}_{1,t}, \Delta \log \hat{Y}_{1,t}; R_t, \pi_{i,t}, \Delta \log L^r_{i,t}, R^L_{i,t}, R^D_{i,t}]\) and measurement equations \(Y_t = [\hat{Y}_{1,t} - \bar{Y}_{1,t}, \bar{Y}_{1,t}; 4\hat{r}_t, \hat{\pi}_{i,t}, \hat{L}^r_{i,t} - \bar{L}^r_{i,t}; 4\hat{r}^L_{i,t}, 4\hat{r}^D_{i,t}]\), where \(\Delta\) denotes the temporal difference operator, \(\bar{X}_t\) is per capita variable of \(X_t\) and \(\hat{x}_t\) is the loglinearized version of \(X_t\). The model matches the data setting \(Y_{\text{obs}} = \bar{Y} + \gamma_t\) where \(\bar{Y}\) is the vector of the mean parameters, we suppose this is a vector of all 0. The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. We compute the posterior moments of the parameters using a sufficiently large number of draws, having made sure that the MCMC algorithm converged. To do this, a sample of 150,000 draws was generated, neglecting the first 50,000. The scale factor was set in order to deliver acceptance rates of between 20 and 30 percent. Convergence was assessed by means of the multivariate convergence statistics taken from Brooks and Gelman (1998).
gesting that the peripheral countries import more than the core countries, implying current account deficits. The estimation of the credit markets openness is interesting, as underlined by Brunnermeier et al. (2012), cross-border banking within the euro area experienced explosive growth, especially after around 2003, helping to fuel property booms in Ireland and southern European countries. The model captures this feature as the degrees of openness of the credit market in peripheral countries is larger than in core countries.

To assess how well the model fits the data, we present in Table 9 and in Figure 7 the second moments of the observable variables and their counterpart in the model. The model does reasonably well in explaining the standard deviation of most of the variables except for some of the financial variables, despite allowing for different degrees of nominal rigidities via the introduction of Calvo contracts and habits. Nevertheless, the model captures well the persistence of all aggregates except for inflation in the core area. Concerning the cross-country correlations, the model does reasonably well in capturing the co-movement of all aggregates, however the model underestimates the cross-country correlation between home and foreign output and investment.

5 Macroprudential Policy

As reported by Lim et al. (2011), a number of instruments may be effective in addressing systemic risks in the financial sector. In this paper, we take into account the classification initially introduced by Blanchard et al. (2013). We assume that macroprudential policy is implemented through two instruments: one directed towards the financial stability of the lender, the other towards the borrower.

5.1 Countercyclical Capital Buffers

First regarding lenders, we assume that macroprudential policy accounts for a ratio related to the Basel I-like capital requirement of the banking system augmented with counter-cyclical capital buffers (Darracq-Pariès et al., 2011),

\[
F_i \left( \frac{BK_{t+1}}{A_{i,t+1}} , ccb_{i,t} \right) = 0.5 \chi_i^{KR} \left( \frac{BK_{t+1}}{A_{i,t+1}} - ccb_{i,t} \right)^2 ,
\]

where \( F(\cdot) \) implies that the bank must pay a quadratic cost whenever the bank capital to risk weighted assets ratio \( BK_{t+1}/A_{i,t+1} \) moves away from a time-varying optimal target \( ccb_{i,t} \). Since Basel III uses the gap between the credit-to-GDP ratio

\[\text{According to the Basel I accords, the banks assets are weighted according to their risk. Letting} \gamma_r^L \text{ and } \gamma_r^B \text{ denote the risk weighting of the Basel I regulatory framework, the risk-weighted}\]
and its long-term trend as a guide for setting countercyclical capital buffers\textsuperscript{12}, we introduce time-varying capital requirements with a target bank capital ratio implemented in country $i$ that follows a rule of the form,

$$ccb_{i,t} = \left(\frac{ctg_{u,t}}{ctg_u}\right)^{\phi^{CCB}_i},$$

(2)

where $ctg_{u,t}$ is the credit-to-GDP ratio of the monetary union $ctg_{u,t} = (ctg_{u,t})^n (ctg_{f,t})^{1-n}$ with $ctg_{u,t} = (L^s_{i,t} + IB^s_{i,t})/Y_{i,t}$ and $\phi^{CCB}_i$ denotes the reaction degree of countercyclical capital buffers to a deviation from the long term credit-to-GDP ratio of the monetary union. Under this setting, the borrower oriented instrument is based on a common objective (the credit to GDP) but allows for some heterogeneity in the way capital buffers are released, such that $\phi^{cb}_e \neq \phi^{cb}_p$.

### 5.2 Loan-to-Income Ratio

Second, regarding borrowers, we assume that macroprudential policy accounts for the evolution of the loan-to-income ratio of firms (Gelain et al., 2012). The rule of loan-to-income is defined by,

$$LTI_{i,t} = \left(\frac{L^d_{i,t+1}/Y_{i,t}}{L^d_{i}/Y_{i}}\right)^{\phi^{LTI}_i},$$

(3)

where $L^d_{i,t+1}$ denotes the lending demand from the home private sector and $Y_{i,t}$ is the output. We assume that Thus when $LTI_{i,t}$ increases, it is interpreted by macroprudential authorities as an excessive growth to credit in comparison to activity.

As noticed, in these two expressions ((1)) and ((3)), the value of loans that is taken into account differs in the two instruments, given the possibility of national banks to engage in cross border lending. Thus, the instrument directed toward the lending side of the economy accounts for the supply of loans in the country, while the instrument directed toward the borrowing side accounts for the fact that national agents can borrow from different national sources. However, neglecting the possibility for banks to engage in cross-border activities would make both instruments substitutable in the evaluation of systemic risk at the national level.

assets for illiquid banks are defined by $A^w_{i,t+1} = \gamma^L_{rw} L^s_{i,t} + (1 - \gamma^L_{rw}) L^t_{i,t}$, while for liquid banks it is determined by $A^w_{i,t+1} = \gamma^L_{rw} L^s_{i,t} + (1 - \gamma^L_{rw}) L^t_{i,t} + \gamma^{IB}_{rw} IB^s_{i,t} + (1 - \gamma^{IB}_{rw}) 7B^s_{i,t}$. Following the Basel accords (2004), interbank loans are given a (fixed) risk weight of 20 percent ($\gamma^{IB}_{rw} = 0.20$), whereas the risk weight attached to corporate loans is 100 percent ($\gamma^L_{rw} = 1$).

\textsuperscript{12}See Drehmann and Tsatsaronis (2014) for further discussions about the credit to GDP ratio as the drivers of countercyclical buffers.
In this set-up, macroprudential instruments affect the general equilibrium of the model through the lending conditions of commercial banks. A tightening of credit conditions due to macroprudential measures will thus increase the interest rate faced by borrowers. In the presence of macroprudential regulations, the corporate loan marginal cost of liquid banks is affected by the macroprudential policy,

\[ 1 + MC_{i,t}^{ill} = \left[ (1 + P_{i,t}^{IB}) + F_{i,t}^{L}BK_{i,t+1}^{ill} \right] \frac{LTI_{i,t}}{1 - \mu^{B}E_{i,t}^{F}}, \]

as well as the corporate loan marginal cost of liquid banks,

\[ 1 + MC_{i,t}^{liq} = \left[ (1 + R_{i,t}) + F_{i,t}^{L}BK_{i,t+1}^{liq} \right] \frac{LTI_{i,t}}{1 - \mu^{B}E_{i,t}^{F}}, \]

and the interbank rate are affected by macroprudential policy,

\[ 1 + R_{i,t}^{IB} = (1 + R_{i,t}) + AC_{i,t}^{IB} + F_{i,t}^{IB}BK_{i,t+1}^{liq}, \]

where \( F_{L} \) and \( F_{IB}^{T} \) denote the derivative of \( F(\cdot) \) in \( L_{i,t}^{s} \) and \( IB_{i,t}^{s} \).

6 The setting of instruments

This section discusses how macroprudential policy should be conducted in the monetary union when taking into account the possibility for countries to adopt heterogeneous practices. We successively consider the consequences of choosing symmetric/asymmetric individual instruments and the consequences of implementing the macroprudential policy using multiple instruments at the regional level. We report in four different tables (Table 2, Table 3, Table 5 and Table 6) the households unconditional consumption gains or losses from leaving the benchmark situation where the European Central Bank implements an optimal monetary policy rule.

In the quantitative simulation, we first search for weights attached to inflation \( \phi^{\pi} \) and GDP growth \( \phi^{\Delta y} \) in the Taylor rule that gives the highest unconditional welfare of households. Here, we maintain the autoregressive parameter of the policy rule \( \rho^{R} \) at its estimated value since it has low effects on welfare. Based on the grid search by 0.01 unit, we limit our attention to policy coefficients in the interval \((1, 3]\) for \( \phi^{\pi} \), \([0, 3]\) for \( \phi^{\Delta y} \) as Schmitt-Grohé and Uribe (2007), and in the interval \([0, 30]\) for macroprudential instruments \( \phi^{CCB}_{i} \) and \( \phi^{LTI}_{i} \). Our grid search interval for macroprudential instruments is consistent with the findings of Gerali et al. (2010). The size of this interval is arbitrary. However, policy coefficients larger than 30 would be difficult to communicate to policymakers or the public (Schmitt-Grohé and Uribe (2007)).
6.1 Setting individual instruments

The indicative list of macroprudential instruments selected by the ESRB provides the member countries with a wide selection of macroprudential measures tailored to address its financial problems. In this subsection, we concentrate on the implementation of single instrument macroprudential policy measures at the regional level.

We report in Table 2 four possibilities of combining macroprudential instruments depending on the adoption of either a lender or a borrower instrument in each part of the Eurozone. Payoffs are defined in terms of permanent consumption gains. In all cases the implementation of macroprudential policy leads to welfare gains at the federal level. However, the macroprudential stance in each region (the values of $\phi_i^{CCB}$ and $\varphi_i^{LTI}$) depends on the instrument adopted in the other part of the monetary union and may imply a regional decrease in welfare with respect to the benchmark situation.

First, the degree of symmetry in the choice of instruments affects national outcomes. Symmetric practices (in regimes (a) and (b)) lead to welfare gains in both parts of the monetary union, while the asymmetric choices of instruments in regimes (b) and (c) leads to welfare losses in the region implementing macroprudential policy using the a lender instrument. In contrast, the region adopting the borrower instruments records very high permanent consumption gains.

Second, leaving member countries choosing the macroprudential instrument (i.e., computing the Nash equilibrium) we find that regions are naturally encouraged to select regime (d) as the equilibrium. In this situation, they adopt symmetrical borrower instruments, as both countries experience welfare gains. However, on federal ground this situation is not optimal, as the average union wide permanent consumption increase ($W_u = 0.382\%$) is much lower than the one encountered in regime (b) ($W_u = 0.785\%$).

Third, The optimal federal situation reached in (b) requires an asymmetric choice of instruments between regions of the Eurozone: the periphery should select a borrower instrument, while the core should select a lender instrument. This makes sense regarding to the fact that peripheral countries are net importers of loans in the Eurozone. However, while the average union welfare increase represents $W_u = 0.785 \%$ of permanent consumption, it leads to high inequalities between regions: the periphery is a net winner (with a permanent consumption increase of $W_p = 3.443 \%$), while welfare decreases in the core (with a permanent consumption decrease of $W_c = -2.061 \%$). This optimal equilibrium is thus difficult to implement, as the core may disagree to participate to the macroprudential scheme with the peripheral countries.

Fourth, another equilibrium should be considered, combining the highest possible federal permanent consumption increase subject to positive welfare gains in
both regions. Under this simple criterion, the best possible outcome is regime (a), i.e., the adoption of symmetric macroprudential measures using lender instruments. In this situation the increase of welfare represents $W_c = 0.301\%$ of permanent consumption in the core countries, $W_p = 1.006\%$ of permanent consumption in the core countries, leading to an average increase of permanent consumption of $W_u = 0.599$ in the Eurozone. However, to be enforceable this equilibrium requires a supranational coordination mechanism such as the ESRB. This situation is more interesting than the Nash equilibrium (d), for peripheral countries and less interesting for core countries.

### 6.2 Combining multiple instruments

The previous subsection has outlined the federal optimality of macroprudential policy using asymmetric instruments. However, symmetric practices should be preferred on national grounds, as they lead to permanent consumption increases in both parts of the Eurozone. A simple way to reconcile these two sets of results is to combine both instruments in each region of the Eurozone. The use of multiple instruments at the regional level may be helpful to address the various natures of financial imbalances, while the symmetric implementation of macroprudential instruments among the members of the monetary union increases both region welfare. This approach is in line with Lim et al (2011) that report many country experiences based on a combination of several instruments while the use of a single instrument to address systemic risk is rare. The rationale for using multiple instruments is to provide a greater assurance of effectiveness by tackling a risk from various angles.

First, the cooperative solution based on the regional use of multiple instruments is reported as regime (e) in Table 3. As observed, welfare increase at the federal level ($W_u = 0.866$) is higher than the one previously reached with the asymmetric implementation of a single instrument ($W_u = 0.785$). Remarkably, each part of the monetary union gets welfare gains, even if the periphery situation ($W_p = 1.813$) clearly improves with respect to the situation of the core countries ($W_c = 0.167$).

The interest of adopting a granular determination of the value of the instruments can be underlined by contrasting the figures obtained in the regime (e) with the ones obtained by imposing a uniform value to the parameters. In the latter situation, the figures are obtained as follows: $W_u = 0.005$, $W_c = -0.146$, $W_p = 0.21$ with instrument set uniformly as, $\phi^{CCB}_c = \phi^{CCB}_p = 3.58$ and $\varphi^{LTI}_c = \varphi^{LTI}_p = 0$. In this situation, the federal welfare gains are negligible while the center incur losses (instruments are set too high) and the welfare gains of the periphery are limited (as the instruments are set too low).

Second, we evaluate the interest of creating a supranational mechanism such as the ESRB to promote the coordination between countries belonging to the
Table 2: Nash Equilibrium Payoffs Matrix in terms of unconditional consumption gains from leaving the standard optimal monetary policy.

<table>
<thead>
<tr>
<th>CORE</th>
<th>PERIPHERY</th>
<th>Lender Tool $\phi_{c}^{CCB} \geq 0$</th>
<th>Borrower Tool $\varphi_{p}^{LTI} \geq 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lender Tool $\phi_{c}^{CCB} \geq 0$</td>
<td>$\mathcal{W}<em>{c} = 0.301, \quad \mathcal{W}</em>{p} = 1.006$</td>
<td>$\mathcal{W}<em>{c} = -0.197, \quad \mathcal{W}</em>{p} = 2.111$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathcal{W}_{u} = 0.599$</td>
<td>$\mathcal{W}_{u} = 0.785$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\phi_{c}^{CCB} = 7.78, \phi_{p}^{CCB} = 5.42$ (a)</td>
<td>$\phi_{c}^{CCB} = 9.22, \varphi_{p}^{LTI} = 0.02$ (b)</td>
<td></td>
</tr>
<tr>
<td>Borrower Tool $\varphi_{c}^{LTI} \geq 0$</td>
<td>$\mathcal{W}<em>{c} = 1.64, \quad \mathcal{W}</em>{p} = -1.919$</td>
<td>$\mathcal{W}<em>{c} = 0.568, \quad \mathcal{W}</em>{p} = 0.126$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathcal{W}_{u} = 0.156$</td>
<td>$\mathcal{W}_{u} = 0.382$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\varphi_{c}^{LTI} = 0.02, \varphi_{p}^{CCB} = 6.76$ (c)</td>
<td>$\varphi_{c}^{LTI} = 0, \varphi_{p}^{LTI} = 0.04$ (d)</td>
<td></td>
</tr>
</tbody>
</table>

Note: best outcome is (b), nash equilibrium is (d)

Table 3: Nash Equilibrium Payoffs Matrix (in terms of unconditional consumption gains from leaving the simple optimal policy)

<table>
<thead>
<tr>
<th>CORE</th>
<th>PERIPHERY</th>
<th>Coordination $\phi_{c}^{CCB} = 2.52, \varphi_{p}^{LTI} = 0.03$</th>
<th>No Coordination $\phi_{c}^{CCB} = 0, \varphi_{p}^{LTI} = 0.01$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coordination $\phi_{c}^{CCB} = 10, \varphi_{c}^{LTI} = 0$</td>
<td>$\mathcal{W}<em>{c} = 0.167, \quad \mathcal{W}</em>{p} = 1.813$</td>
<td>$\mathcal{W}<em>{c} = -2.061, \quad \mathcal{W}</em>{p} = 3.443$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathcal{W}_{u} = 0.866$</td>
<td>$\mathcal{W}_{u} = 0.307$ (f)</td>
<td></td>
</tr>
<tr>
<td>No Coordination $\phi_{c}^{CCB} = 1.77, \varphi_{c}^{LTI} = 0.03$</td>
<td>$\mathcal{W}<em>{c} = 1.176, \quad \mathcal{W}</em>{p} = -0.586$</td>
<td>$\mathcal{W}<em>{c} = 0.308, \quad \mathcal{W}</em>{p} = -0.386$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\mathcal{W}_{u} = 0.437$ (g)</td>
<td>$\mathcal{W}_{u} = 0.015$ (h)</td>
<td></td>
</tr>
</tbody>
</table>

Note: best outcome is (e), nash equilibrium is (h)
monetary union by studying the enforceability of the optimal cooperative solution with a Nash bargaining game (Nash Jr (1950, 1953)). Following the bargaining theory, we examine how the surplus generated by macroprudential policy will be split between EMU participants when national governments may have an incentive to set unilaterally macroprudential instruments. Here, there are two participants in the game \( i = c, p \). Each player considers two possible strategies: *coordination* and *no coordination*. If both players choose to coordinate macroprudential instruments, they maximize the welfare index of the monetary union \( W_u \). If one country \( i \) chooses to deviate singly, it maximizes the welfare index of its country \( W_i \) to the detriment of the monetary union. In this situation the existence of an enforceability mechanism is necessary to reach the cooperative equilibrium, as each country may have an incentive to deviate from the cooperative equilibrium. Indeed, as reported in the table, the cooperative equilibrium does not coincide with the Nash equilibrium: i.e., the best situation can not be reached without an external mechanism such as the ESRB.

As reported in Table 3, we find that each country would choose to follow the non cooperative strategy because incentives to deviate from the cooperative equilibrium are too large. Leaving countries to choose macroprudential governance with multiple instruments set regionally would lead to regime (h). In this situation the increase of welfare represents \( W_c = 0.308\% \) of permanent consumption in the core countries, the decrease of welfare in the periphery represents \( W_p = -0.386\% \) of permanent consumption, both figures leading to a slight average increase of permanent consumption of \( W_u = 0.015\% \), far away from the cooperative outcome of regime (e).

In summary, as already underlined in the case of a single instrument, the Pareto optimal equilibrium cannot be reach on the basis of national incentives. It would thus be necessary to enforce cooperation through federal institutional incentives, such as the one developed through the ESRB initiative.

### 6.3 Macroeconomic performances

Table 4 reports the macroeconomic performances of macroprudential policy regimes presented above. We more particularly concentrate our analysis on the consequences of the macroprudential policy on the standard deviation of activity, inflation and the credit to GDP ratio (measuring the evolution of financial imbalances) for the monetary union and for each region.

First, looking at federal figures, we observe that the organization of macroprudential policy has only a marginal impact on both the standard deviation of inflation and activity. In contrast, the choice of the macroprudential regime clearly affects the standard deviation of the credit to GDP ratio. In particular, this standard deviation (reaching 4.26 without macroprudential measures) can falls to 2.72
under the optimal regime (e) combining two macroprudential instruments in both regions.

Second, turning to regional figures, the model reports high heterogeneities regarding both macroeconomic and financial indicators. Overall the core countries are characterized by less instability (the three standard deviations are much lower) than peripheral countries. On average the standard deviation of inflation is one third of the peripheral figure, while the standard deviation of output is around 20% less than the peripheral figure. The choice of macroprudential instruments affects this macroeconomic heterogeneity.

Third, the most striking heterogeneity between regions is reported for the regional standard deviation of the credit to GDP ratio. This standard deviation moves from 2.72 (without macroprudential measures) to 1.59 (in the optimal regime (e)) for the core countries, while it fluctuates sharply between 7.37 (regime (f)) to 3.14 (regime (c)). Noticeably, the lowest value for the standard deviation of the credit to growth ratio does not correspond to the optimal policy regime nor does the highest value corresponds to the optimal monetary policy situation. This feature can be explained by the fact that the criterion used to reach the optimality of the macroprudential policy accounts for the interdependence of regions in the currency union. However, the standard deviation is quite low in the periphery under the optimal macroprudential regime (e).

To understand why the optimal situation in terms of welfare does not correspond to the situation with the best macroeconomic record, one has to remember that welfare depends on consumption and labour effort. In particular, the consumption smoothing component is a key aspect of welfare. However, the Pareto optimal situation leads to federal macroeconomic results that are rather good as the value for the three standard deviation is rather low. Nevertheless, as observed the lowest value for the standard deviation of the credit to growth ratio is obtained for regime (g) which is not the optimal one.
**Indicative list of macro-prudential instruments**

1. **Mitigate and prevent excessive credit growth and leverage:**
   - Counter-cyclical capital buffer
   - Sectoral capital requirements (including intra-financial system)
   - Macro-prudential leverage ratio
   - Loan-to-value requirements (LTV)
   - Loan-to-income/debt (service)-to-income requirements (LTI)

2. **Mitigate and prevent excessive maturity mismatch and market illiquidity:**
   - Macro-prudential adjustment to liquidity ratio (e.g. liquidity coverage ratio)
   - Macro-prudential restrictions on funding sources (e.g. net stable funding ratio)
   - Macro-prudential unweighted limit to less stable funding (e.g. loan-to-deposit ratio)
   - Margin and haircut requirements

3. **Limit direct and indirect exposure concentration:**
   - Large exposure restrictions
   - CCP clearing requirement

4. **Limit the systemic impact of misaligned incentives with a view to reducing moral hazard:**
   - SIFI capital surcharges

5. **Strengthen the resilience of financial infrastructures:**
   - Margin and haircut requirements on CCP clearing
   - Increased disclosure
   - Structural systemic risk buffer

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Figure 3: Recommendation of the European Systemic Risk Board of 4 April 2013 on intermediate objectives and instruments of macro-prudential policy (ESRB/2013/1)
Figure 4: The model of a two-country monetary union with international bank loan flows

Figure 5: Observable variables
<table>
<thead>
<tr>
<th></th>
<th>Monetary Union</th>
<th></th>
<th>Core</th>
<th>Periphery</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$sd(\pi_u)$</td>
<td>$sd(y_u)$</td>
<td>$sd(\Delta ctg_u)$</td>
<td>$sd(\pi_c)$</td>
</tr>
<tr>
<td>Optimal</td>
<td>0.10</td>
<td>0.8</td>
<td>4.26</td>
<td>0.06</td>
</tr>
<tr>
<td>Empirical</td>
<td>0.11</td>
<td>0.86</td>
<td>4.22</td>
<td>0.06</td>
</tr>
</tbody>
</table>

*with individual instruments:*

(a) | 0.10 | 0.8 | 3.15 | 0.06 | 0.71 | 1.93 | 0.16 | 0.93 | 4.83 |
(b) | 0.11 | 0.8 | 3.51 | 0.06 | 0.71 | 1.64 | 0.17 | 0.93 | 6.09 |
(c) | 0.10 | 0.79| 2.79 | 0.06 | 0.71 | 2.53 | 0.15 | 0.91 | 3.14 |
(d) | 0.10 | 0.79| 3.18 | 0.06 | 0.70 | 2.68 | 0.16 | 0.91 | 3.87 |

*with combined instruments:*

(e) | 0.10 | 0.80| 2.72 | 0.05 | 0.71 | 1.59 | 0.16 | 0.92 | 4.29 |
(f) | 0.11 | 0.81| 4.08 | 0.06 | 0.72 | 1.7  | 0.17 | 0.94 | 7.37 |
(g) | 0.10 | 0.79| 2.66 | 0.06 | 0.71 | 2.01 | 0.16 | 0.91 | 3.56 |
(h) | 0.10 | 0.8 | 3.61 | 0.06 | 0.71 | 1.95 | 0.16 | 0.92 | 5.9  |

Table 4: Macroeconomic performances of different policy regimes
7 Sensitivity Analysis

In this last section we evaluate the sensitivity of the previous results with a key assumption of the model, namely the possibility of banks to engage in cross border interbank and corporate loans. We more particularly concentrate on the welfare implication related to international financial spillovers by considering a situation where banks do not engage in cross border loans (i.e. we impose, $\alpha^L_i = \alpha^B_i = 0$ in the model).

As reported in Table 5, if each part of the monetary union selects one instrument, the outcome regarding the symmetric/asymmetric choice of the macroprudential tool previously underlined becomes irrelevant with banking autarky, as regional outcomes are positive across all policy regimes (a)-(d). As there is no spillover related to cross border lending, the choice of the macroprudential policy of the other region is clearly nationally less painful in the case of the choice of asymmetric instruments. However, as reported, the choice of the instrument in the core countries has a deep impact on national and federal welfare. The adoption of the lender instrument should be preferred as it leads to a much higher increase in permanent consumption growth. In contrast the choice of the instrument in the peripheral countries has only a very low impact on the final result.

Remarkably, the optimal policy regime on federal grounds is still regime (b). In this situation core countries select an instrument to target the banking sector, while peripheral countries select the instrument oriented towards the borrowers. However, cross-border loan flows are a critical feature to address cooperation incentives issues, as we observe a Nash equilibrium reversal in banking autarky. Now, the Nash equilibrium corresponds to the optimal choice of instruments on union wide welfare. National incentives lead to the optimal outcome: as each country is able to target the origin of regional imbalances (i.e., impose constrains on either lenders or borrowers) without creating spillovers to the other part of the monetary union, there is no need to homogenize instruments as we previously found with cross border lending. Furthermore, as the Nash and optimal equilibria are the same, there is no need to create an exogenous mechanism to reach the optimal equilibrium as long as each part of the monetary union can select a well tailored macroprudential instrument. This result is similar to the one obtained by Quint and Rabanal (2013) in a model ignoring cross border lending, showing that there was no gain to get from the coordination of macroprudential instruments between the members of a monetary union.

In Table 6, we report results when both instruments are combined in each region of the Eurozone. As previously noticed for cross border lending, the use of multiple instruments in each region increases welfare in the monetary union. The welfare increase now represents 0.902% of permanent consumption instead of 0.884% when using a single instrument. Furthermore, the core has less incentive to
deviate from the coordinated scheme, as it is neutral with respect to this choice. The picture is different for the periphery as peripheral countries clearly favour the non cooperation scheme. Thus combining individual strategies makes the equilibrium indeterminate as it can lie either on regimes (f) and (h).

However, in both regimes (f) and (h) the average union wide permanent consumption increase is lower than the one encountered in the optimal regime (e). Thus cooperation still requires the use of a supranational enforcing mechanism such as the one represented by the ESRB. However, the difference in the value of permanent consumption increases is not as high as the one reported for the situation with cross border lending. As there is no spillover related to cross border lending, there is less loss in welfare increase related to the deviation from the optimal cooperative equilibrium. Thus, implementing uncoordinated regional macroprudential policy without cross border lending is less painful, as there is less spillovers that may dilute the effect of unappropriated macroprudential decisions in other side of the monetary union.

8 Conclusion

This paper has discussed the degree of homogeneity to be imposed to macroprudential measures for countries belonging to the Eurozone. Assessing the fact that the implementation of the macroprudential mandate has been set on cooperative decisions, we have developed a two-country DSGE model that accounts for some major features of the Eurozone (such as the key role of the banking system, cross-border bank loans or the heterogeneity in financial factors) to provide a quantitative evaluation of welfare gains coming from a more symmetric treatment of countries in the implementation of macroprudential measures. We have separated the Eurozone in two parts (the core and the periphery) and evaluated how a macroprudential policy combining two instruments (namely, bank capital requirements and the evolution of the loan-to-income ratio) should be implemented.

Our main conclusions underline a possible conflict between the federal and the national levels in the implementation of heterogenous macroprudential measures based on a single instrument as the Pareto optimal situation computed on a federal ground requires an asymmetric choice of instruments, while more symmetric practices should be preferred on regional ground. Second, The adoption of combined instruments in each country solves this potential conflict as it leads to both a higher welfare increase in the Pareto optimal equilibrium and always incurs national welfare gains. However, in all cases, the Pareto optimal equilibrium cannot be reached on national incentives and requires a supranational enforcing mechanism.

These elements should be useful to analyze the proposed evolution of macropru-
Table 5: Nash Equilibrium Payoffs Matrix (in terms of unconditional consumption gains from leaving the simple optimal policy)

<table>
<thead>
<tr>
<th>CORE</th>
<th>PERIPHERY</th>
<th>Borrower Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lender Tool</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi_c^{CCB} \geq 0 )</td>
<td>( \phi_p^{CCB} = 10, \phi_c^{CCB} = 2.94 )</td>
</tr>
<tr>
<td></td>
<td>( W_c = 0.877, \ W_p = 0.719 ) ( W_u = 0.811 )</td>
<td>( W_c = 0.940, \ W_p = 0.317 ) ( W_u = 0.157 )</td>
</tr>
<tr>
<td></td>
<td>( \phi_c^{LTI} \geq 0 )</td>
<td>( \phi_c^{LTI} = 0.01, \phi_p^{CCB} = 4.37 )</td>
</tr>
</tbody>
</table>

Note: best outcome and nash equilibrium are (b)

Table 6: Nash Equilibrium Payoffs Matrix (in terms of unconditional consumption gains from leaving the simple optimal policy)

<table>
<thead>
<tr>
<th>CORE</th>
<th>PERIPHERY</th>
<th>Borrower Tool</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coordination</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \phi_c^{CCB} = 10, \phi_c^{LTI} = 0 )</td>
<td>( \phi_p^{CCB} = 1.29, \phi_c^{LTI} = 0.03 )</td>
</tr>
<tr>
<td></td>
<td>( W_c = 0.94, \ W_p = 0.849 ) ( W_u = 0.902 )</td>
<td>( W_c = 0.94, \ W_p = 0.849 ) ( W_u = 0.902 )</td>
</tr>
<tr>
<td></td>
<td>( \phi_c^{CCB} = 10, \phi_c^{LTI} = 0.00 )</td>
<td>( \phi_c^{LTI} = 0.01, \phi_p^{CCB} = 4.37 )</td>
</tr>
</tbody>
</table>

Note: best outcome is (e) and (g), nash is (h) and (f)
ential policy organization in the Eurozone. There are important welfare gains in implementing national adjusted macroprudential measures, but a federal institution is necessary to oblige EU members to cooperate. Without a federal constraint on macroprudential policy implementation, the Euro members may choose not to coordinate which may significantly reduce the welfare of the Eurosystem.
References


Medina, J. P., Roldós, J., 2013. Monetary and macroprudential policies to manage capital flows. forthcoming IMF/WP.


A Model Derivations

As shown in Figure 4, the two countries share a common currency but are not equal in size. Our model describes a monetary union made of two asymmetric countries $i \in \{c, p\}$ (where $c$ is for Core and $p$ for periphery) of relative sizes $n_c$ and $n_p$ inspired by (Kolasa, 2009). Each country is populated by households that consume save and supply labor. Intermediate firms supply differentiated goods in a monopolistically competitive market and set prices in a staggered basis. Final producers are CES packers, they aggregate the differentiated goods from intermediate firms and sell it to households and capital producers. Capital producers recycle the used capital stock and invest new capitals. Entrepreneurs buy capitals from capital producers and sell it to intermediate firms. Entrepreneurs are credit constrained, they borrow funds to the banking system. Banks provide loans to entrepreneurs and deposit services in a monopolistically competitive market and set interest rate in staggered contracts. Banks are not homogenous, they are either liquid or illiquid according to their access to ECB fundings leading to an international interbank market where liquid banks provides interbanks loans to both home and foreign banks.

We aggregate households, firms, entrepreneurs and banks using the following aggregator for agent $x \in [0,1]$,

$$G(X_{i,t}) = \begin{cases} \int_0^{n_c} X_{i,t}(x) \, dx & \text{for } i = c \\ \int_{n_c}^{n_c+n_p} X_{i,t}(x) \, dx & \text{for } i = p \end{cases} \quad (A.1)$$

A.1 The households utility maximization problem

In each economy there is a continuum of identical households who consume, save and work in intermediate firms. The total number of households is normalized to 1. The representative household $j \in [0,1]$ maximizes the welfare index,

$$\max_{\{C_{i,t}(j), H_{i,t}(j), D_{i,t+1}(j)\}} \mathbb{E}_t \sum_{\tau=0}^\infty \beta^\tau e^{d_{i,t+r}} \left( \frac{(C_{i,t+r}(j) - h_i^c C_{i,t-1+r})^{1-\sigma^c}}{1 - \sigma^c} - \chi_i \frac{H_{i,t+r}(j)^{1+\sigma^L}}{1 + \sigma^L} \right),$$

subject to,

$$\frac{W^h_{i,t}}{P^C_{i,t}} H_{i,t}(j) + (1 + R_{i,t-1}^D) \left( D_{i,t}(j) + \Pi_{i,t}^p(j) + M_i(j) \right) = C_{i,t}(j) + \frac{D_{i,t+1}^D(j)}{P^C_{i,t}} + \frac{T_{i,t}(j)}{P^C_{i,t}} + \frac{P_{i,t}^C AC_{i,t}(j)}{P^C_{i,t}} \quad (A.2)$$
<table>
<thead>
<tr>
<th></th>
<th>Prior distributions</th>
<th>Posterior distribution [5%:95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shape</td>
<td>Mean</td>
</tr>
<tr>
<td><strong>STANDARD DEVIATION</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity</td>
<td>$\sigma^A_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Spending</td>
<td>$\sigma^G_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Preferences</td>
<td>$\sigma^U_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Investment cost</td>
<td>$\sigma^I_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Firms cost-push</td>
<td>$\sigma^P_t$</td>
<td>$\mathcal{IG}$</td>
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<tr>
<td>Wage cost-push</td>
<td>$\sigma^W_t$</td>
<td>$\mathcal{IG}$</td>
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<tr>
<td>Collateral crunch</td>
<td>$\sigma^N_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Deposit cost-push</td>
<td>$\sigma^D_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Bank liabilities</td>
<td>$\sigma^E_t$</td>
<td>$\mathcal{IG}$</td>
</tr>
<tr>
<td>Loans cost-push</td>
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<td>$\mathcal{IG}$</td>
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<td>Monetary Policy</td>
<td>$\sigma^R_t$</td>
<td>$\mathcal{IG}$</td>
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<td><strong>AR(1) ROOT</strong></td>
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<td>Productivity</td>
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<td>Preferences</td>
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<td>$\mathcal{B}$</td>
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<tr>
<td>Firms cost-push</td>
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<tr>
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<tr>
<td>Collateral crunch</td>
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<td>$\mathcal{B}$</td>
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<tr>
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<td>$\mathcal{N}$</td>
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<tr>
<td>Spending-productivity</td>
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<td>$\mathcal{N}$</td>
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</tbody>
</table>

Table 7: Prior and Posterior distributions of shock processes
Table 8: Prior and Posterior distributions of structural parameters and shock processes. Note: IG denotes the Inverse Gamma distribution, B the Beta, N the Normal, G the Gamma.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Shape</th>
<th>Mean</th>
<th>Std.</th>
<th>CORE PERIPHERY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption aversion</td>
<td></td>
<td>1.50</td>
<td>0.25</td>
<td>0.92 [0.69:1.13]</td>
</tr>
<tr>
<td>Labour disutility</td>
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<td>2.00</td>
<td>0.75</td>
<td>0.76 [0.29:1.21]</td>
</tr>
<tr>
<td>Consumption inertia</td>
<td></td>
<td>0.70</td>
<td>0.10</td>
<td>0.51 [0.38:0.64]</td>
</tr>
<tr>
<td>Calvo prices</td>
<td></td>
<td>0.75</td>
<td>0.05</td>
<td>0.77 [0.7:0.84]</td>
</tr>
<tr>
<td>Indexation prices</td>
<td></td>
<td>0.50</td>
<td>0.15</td>
<td>0.27 [0.1:0.43]</td>
</tr>
<tr>
<td>Calvo wage</td>
<td></td>
<td>0.75</td>
<td>0.05</td>
<td>0.84 [0.79:0.89]</td>
</tr>
<tr>
<td>Indexation wage</td>
<td></td>
<td>0.50</td>
<td>0.15</td>
<td>0.37 [0.21:0.53]</td>
</tr>
<tr>
<td>Calvo loan rates</td>
<td></td>
<td>0.50</td>
<td>0.10</td>
<td>0.75 [0.68:0.82]</td>
</tr>
<tr>
<td>Calvo deposit rates</td>
<td></td>
<td>0.50</td>
<td>0.10</td>
<td>0.76 [0.73:0.79]</td>
</tr>
<tr>
<td>Investment cost</td>
<td></td>
<td>4.00</td>
<td>1.50</td>
<td>6.37 [4.58:8.23]</td>
</tr>
<tr>
<td>Capital utilization</td>
<td></td>
<td>0.50</td>
<td>0.07</td>
<td>0.53 [0.43:0.63]</td>
</tr>
<tr>
<td>Premium elasticity</td>
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<td>0.05</td>
<td>0.03</td>
<td>0.24 [0.15:0.32]</td>
</tr>
<tr>
<td>Loan demand habits</td>
<td></td>
<td>0.50</td>
<td>0.15</td>
<td>0.74 [0.62:0.87]</td>
</tr>
<tr>
<td>Share of illiquid banks</td>
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<td>0.10</td>
<td>0.38 [0.29:0.46]</td>
</tr>
<tr>
<td>Interbank demand habits</td>
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<td>0.15</td>
<td>0.29 [0.07:0.48]</td>
</tr>
<tr>
<td>Interbank monitoring cost</td>
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<td>0.02</td>
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<td>Capital requirements</td>
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<td>13.3 [9.14:17.37]</td>
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<td>Goods openness</td>
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<td>0.12</td>
<td>0.04</td>
<td>0.08 [0.04:0.12]</td>
</tr>
<tr>
<td>Investment openness</td>
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<td>0.12</td>
<td>0.04</td>
<td>0.05 [0.02:0.07]</td>
</tr>
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<td>Corporate openness</td>
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<td>0.03</td>
<td>0.03 [0.01:0.04]</td>
</tr>
<tr>
<td>Interbank openness</td>
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<td>0.25</td>
<td>0.03 [0.02:0.03]</td>
</tr>
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<td>Goods substitution</td>
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<td>0.50</td>
<td>1.16 [0.73:1.59]</td>
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<td>MPR smoothing</td>
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<td>0.50</td>
<td>0.20</td>
<td>0.85 [0.81:0.88]</td>
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<td>MPR inflation</td>
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<td>2.00</td>
<td>0.15</td>
<td>1.67 [1.42:1.93]</td>
</tr>
<tr>
<td>MPR GDP</td>
<td></td>
<td>0.125</td>
<td>0.05</td>
<td>0.20 [0.13:0.27]</td>
</tr>
</tbody>
</table>

Table 9: Empirical and Theoretical Standard deviations

<table>
<thead>
<tr>
<th></th>
<th>ΔY_{i,t}</th>
<th>ΔC_{i,t}</th>
<th>ΔI_{i,t}</th>
<th>π^{i}_{i,t}</th>
<th>ΔW^{r}_{i,t}</th>
<th>ΔL^{r}_{i,t}</th>
<th>R^{D}_{i,t}</th>
<th>R^{D}_{i,t}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empirical - Home</td>
<td>0.80</td>
<td>0.67</td>
<td>1.68</td>
<td>0.18</td>
<td>0.48</td>
<td>1.22</td>
<td>1.12</td>
<td>0.27</td>
</tr>
<tr>
<td>Theoretical - Home</td>
<td>0.86</td>
<td>0.69</td>
<td>2.61</td>
<td>0.24</td>
<td>0.52</td>
<td>1.15</td>
<td>1.51</td>
<td>0.60</td>
</tr>
<tr>
<td>Empirical - Foreign</td>
<td>0.86</td>
<td>0.93</td>
<td>2.25</td>
<td>0.38</td>
<td>0.72</td>
<td>2.05</td>
<td>1.05</td>
<td>0.44</td>
</tr>
<tr>
<td>Theoretical - Foreign</td>
<td>0.95</td>
<td>0.91</td>
<td>2.87</td>
<td>0.41</td>
<td>0.74</td>
<td>1.73</td>
<td>1.08</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Marginal log-likelihood -1004.2
Here, $C_{i,t}(j)$ is the consumption index, $h_i^C \in [0; 1]$ is a parameter that accounts for consumption habits, $H_{i,t}(j)$ is labour effort, $\varepsilon_{i,t}^U$ is an exogenous $AR(1)$ shock to household preferences. The income of the representative household is made of labour income (with the nominal wage $W_{i,t}$ and the consumption price index $P_{c,t}$), interest payments for deposit services, (where $D_{i,t}^d(j)$ stands for the deposits services supplied by banks and subscribed in period $(t-1)$ and $R_{i,t}^D$ is the nominal rate of deposits between period $t-1$ an period $t$), and earnings from shareholdings (where $\Pi_{i,t}^D(j)$ are the nominal amount of dividends he receives from final good producers). The representative household spends this income on consumption, deposits and tax payments (for a nominal amount of $T_{i,t}(j)$). Finally, we assume that the household has to pay quadratic adjustment costs to buy new deposits, according to the function, $AC^D_{i,t}(j) = \beta(D_{i,t+1} - D_{i,t}(j))^2/\bar{D}_i(j)$, where $\bar{D}_i(j)$ is the steady state level of deposits.

Letting $\lambda_{i,t}$ denotes the Lagrangian multiplier of the household budget balance sheet, the first order conditions that solve this problem can be summarized with an Euler deposit condition,

$$
\beta \left( 1 + R_{i,t}^D \right) = \mathbb{E}_t \left\{ e^{e_{i,t}^U} \frac{P_{c,t+1}^{c}}{e^{e_{i,t+1}^U}} \left( \frac{C_{i,t+1}(j) - h_i^C C_{i,t}}{C_{i,t}(j) - h_i^C C_{i,t-1}} \right)^{\sigma^c} \right\},
$$

(A.4)

and a labour supply function,

$$
\frac{W_{i,t}^h}{P_{c,t}} = \lambda_{i,t} H_{i,t}(j) \left( C_{i,t}(j) - h_i^C C_{i,t-1} \right)^{\sigma^c}
$$

(A.5)

### A.2 Labor Unions

Households provide differentiated labor types, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogenous labor to intermediate firms. Each representative union is related to an household $j \in [0; 1]$. Assuming that the trade union is able to modify its wage with a probability $1 - \theta_i^W$, it chooses the optimal wage $W^*_i, t(j)$ to maximize its expected sum of profits,

$$
\max_{\{W^*_{i,t}(j)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^W)^\tau \frac{\lambda_{i,t+\tau}^C}{\lambda_{i,t}^C} \left[ \frac{W^*_{i,t}(j)}{P_{c,i,t+\tau}} \prod_{k=1}^{\tau} \left( \pi_{c,i,t+k-1}^C \right)^{\xi_{i,t}^W} - \frac{W_{i,t+\tau}^h(j)}{P_{c,i,t+\tau}} \right] H_{i,t+\tau}(j) \right\},
$$

subject to the downgrade sloping demand constraint from labor packers,

$$
H_{i,t+\tau}(j) = \frac{1}{n_i} \left( \frac{W^*_{i,t}(j)}{W_{i,t+\tau} \prod_{k=1}^{\tau} \left( \pi_{c,i,t+k-1}^C \right)^{\xi_{i,t}^W}} \right)^{-\varepsilon^W} H_{i,t+\tau}, \forall \tau > 0,
$$

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where $H_{i,t} (j)$ denotes the quantity of differentiated labor types $j$ that is used in the labor packer production with time-varying substitutability $\varepsilon_W$ between different labor varieties. The first order condition results in the following equation for the real wage,

$$\frac{W_{i,t}^* (j)}{P_{i,t}^c} = \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} \left( \theta_t^W \right)^{\tau} \frac{X_{i,t+\tau}}{X_{i,t}} \frac{\varepsilon_W}{\varepsilon_{W-1}} e^{\varepsilon_{i,t+\tau} W_{i,t+\tau}^h(j) H_{i,t+\tau} (j)} \frac{W_{i,t+\tau} H_{i,t+\tau} (j)}{P_{i,t+\tau}}}{\mathbb{E}_t \sum_{\tau=0}^{\infty} \left( \theta_t^W \right)^{\tau} \frac{X_{i,t+\tau}}{X_{i,t}} \prod_{k=1}^{\tau} \left( \frac{c_{i,t+k-1}}{c_{i,t+k}} \right)^{\xi^W} H_{i,t+\tau} (j)}$$

(A.6)

where $\varepsilon^W_{i,t}$ is an ad hoc cost-push shock to the real wage equation following an AR(1) process.

### A.3 The Final Goods Sector

This sector is populated by two groups of agents: intermediate firms and one final firm. Intermediate firms produce differentiated goods $i$, decide on labour and capital inputs on a perfectly competitive inputs market, and set prices according to the Calvo model. The final good producer act as a consumption bundler by combining national intermediate goods to produce the homogenous final good.

The final good producers is perfectly competitive and maximizes profits subject to the supply curve,

$$\max_{Y_{i,t}} P_{i,t} Y_{i,t} - G (P_{i,t} (i) Y_{i,t} (i))$$

s.t. $Y_{i,t} = \left[ \left( \frac{1}{n_i} \right)^{1/\epsilon_P} G (Y_{i,t} (i)^{\epsilon_P-1}) \right]^{\epsilon_P/(\epsilon_P-1)}$

We find the intermediate demand functions associated with this problem are,

$$Y_{i,t} (i) = \frac{1}{n_i} \left( \frac{P_{i,t} (i)}{P_{i,t}} \right)^{-\epsilon_P} Y_{i,t}^d, \quad \forall i$$

(A.7)

where $Y_{i,t}^d$ is the aggregate demand. Thus the aggregate price index of all varieties in the economy writes,

$$P_{i,t} = \left[ \frac{1}{n_i} G (P_{i,t} (i)^{1-\epsilon_P}) \right]^{1/\epsilon_P}$$

(A.8)

### A.4 The Intermediate Goods Sector

The representative intermediate firm $i$ has the following technology,

$$Y_{i,t} (i) = e^{\varepsilon_{i,t} A} K_{i,t} (i)^\alpha H_{i,t}^d (i)^{1-\alpha}$$

(A.9)
where \( Y_{i,t}(i) \) is the production function of the intermediate good that combines capital \( K_{i,t}(i) \), labour demand \( H_{i,t}^d(i) \) to household and technology \( \varepsilon_{i,t}^A \). Here, \( \varepsilon_{i,t}^A \) is an AR(1) productivity shock.

Intermediate goods producers solve a two-stage problem. In the first stage, taken the input prices \( W_{i,t} \) and \( Z_{i,t} \) as given, firms rent inputs \( H_{i,t}^d(i) \) to household and \( K_{i,t}(i) \) in a perfectly competitive factor market in order to minimize costs subject to the production constraint (A.9). Combining the first order conditions with the production function leads to the real marginal cost expression,

\[
\frac{MC_{i,t}(i)}{P_{i,t}^c} = \frac{MC_{i,t}^c}{P_{i,t}^c} = \frac{1}{\varepsilon_{i,t}^A} \left( \frac{Z_{i,t}}{P_{i,t}^c} \right)^\alpha \left( \frac{W_{i,t}}{P_{i,t}^c} \right)^{(1-\alpha)} (1-\alpha)^{-(1-\alpha)} \alpha^\alpha \tag{A.10}
\]

inputs must also satisfy,

\[
\alpha \frac{W_{i,t}}{P_{i,t}^c} H_{i,t}^d(i) = (1-\alpha) \frac{Z_{i,t}}{P_{i,t}^c} K_{i,t}(i) \tag{A.11}
\]

In the second-stage, firm \( i \) set prices according to a Calvo mechanism, each period firm \( i \) is not allowed to reoptimize its price with probability \( \theta_{i,t}^P \) but price increases of \( \xi_{i,t}^P \in [0; 1] \) at last period’s rate of price inflation, \( P_{i,t}(i) = \pi_{i,t-1}^P P_{i,t-1}(i) \). The final firm allowed to modify its selling price with a probability \( 1 - \theta_{i,t}^P \) chooses \( \{ P_{*t}^s(i) \} \) to maximize its expected sum of profits,

\[
\max_{\{ P_{*t}^s(i) \}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \left( \theta_{i,t}^P \beta \right)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \left[ P_{*t}^s(i) \prod_{k=1}^{\tau} \pi_{i,t+k-1}^P \right] Y_{i,t+\tau}(i) \right\},
\]

under the demand constraint from final goods producers,

\[
Y_{i,t+\tau}(i) = \frac{1}{n_i} \left( \frac{P_{*t}^s(i)}{P_{i,t+\tau}} \prod_{k=1}^{\tau} \pi_{i,t+k-1}^P \right) ^{-\varepsilon_{i,t}^P} Y_{i,t+\tau}^d, \forall \tau > 0
\]

where \( Y_{i,t}^d \) represents the quantity of the goods demanded in country \( i \) and \( \lambda_{i,t}^c \) is the household marginal utility of consumption. The first order condition that defines the price of the representative firm \( i \) is,

\[
P_{*t}^s(i) = \frac{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \left( \theta_{i,t}^P \beta \right)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \varepsilon_{i,t}^P MC_{i,t+k} Y_{i,t+\tau}(i) \right\}}{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \left( \theta_{i,t}^P \beta \right)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \pi_{i,t+k-1}^P Y_{i,t+\tau}(i) \right\}} \tag{A.12}
\]

where \( \varepsilon_{i,t}^P \) is an ad hoc cost-push shock to the inflation equation following an AR(1) process.
A.5 Entrepreneurs

Each intermediate firm hires labour freely, but requires funds to finance the renting of capital needed to produce the intermediate good. The amount of capital to be financed by the representative entrepreneur is equal to $Q_{i,t}K_{i,t+1}(e)$, where $Q_{i,t}$ is the price of capital. This quantity is financed by two means: the net wealth of entrepreneur $e$, $N_{i,t}(e)$, and the amount that is borrowed by the entrepreneur from the banking system, $L_{i,t+1}(e)$.

A.5.1 The Balance Sheet

The entrepreneur balance sheet writes,

$$Q_{i,t}K_{i,t+1}(e) = L^H_{i,t+1}(e) + N_{i,t+1}(e) \quad (A.13)$$

where $L^H_{i,t+1}(e)$ stands for lending demand with external habits $h^L_i$ to fit the data implying that,

$$L^H_{i,t+1}(e) = L_{i,t+1}(e) - h^L_i (L_{i,t} - \bar{L}_i)$$

Concerning these external habits, $L^H_{i,t+1}(e)$, when there is no habits such that $h^L_i = 0$, then $L^H_{i,t+1}(e) = L_{i,t+1}(e)$, thus loans habits disappears in steady state $L^H_i(e) = \bar{L}_i(e)$.

A.5.2 The Distribution of Risky Investment Projects

We assume that each entrepreneur $e \in [0,1]$ conducts a mass $\omega \in [\omega_{\min}, +\infty)$ of heterogeneous investment projects, they are risky so that some of the projects will have negative profits. To model individual riskiness, we assume that the aggregate return of investment projects $R^k_{i,t}$ is multiplied by a random value $\omega$, so that the net return of its individual project is, $\omega R^k_{i,t}$. Since he must repay to the bank $L^H_{i,t+1}(e)$ given a borrowing rate $P^L_{i,t}(e)$, the net profit of the project $\omega$ is $\omega R^k_{i,t}Q_{i,t+1}K_{i,t}(e, \omega) - P^L_{i,t-1}(e) L^H_{i,t}(e, \omega)$. To separate profitable investment project from non-profitable ones, there exists a critical value (a cutoff point) defined as $\omega^{C}_{i,t}(e)$ such that the project just breaks even. Thereby the threshold is computed by,

$$\omega^{C}_{i,t}(e) R^k_{i,t}Q_{i,t-1}K_{i,t}(e, \omega^{C}_{i,t}) = P^L_{i,t-1}(e) L^H_{i,t}(e, \omega^{C}_{i,t}) \quad (A.14)$$

We assume that the level of the individual profitability affects the survival of the entrepreneur:

- for a high realization of the $\omega$, namely $\omega \geq \omega^{C}_{i,t}(e)$, the entrepreneur’s $\omega^{th}$ project is profitable;
for a low realization of \( \omega \), namely \( \omega < \omega_{t,t}^C \), the entrepreneur’s \( \omega^{th} \) project is not gainful, and he does not make any repayment to the banking system.

By assuming that entrepreneurs projects are drawn from a Pareto distribution, then \( \omega \sim \mathcal{P}(\kappa, \omega_{\min}) \) where \( \omega \in [\omega_{\min}, +\infty) \), \( \kappa \) is the shape parameter and \( \omega_{\min} \) is the minimum bound of \( \omega \). Aggregating the financial contracts, we define the conditional expectation of \( \omega \) when entrepreneur’s project is gainful by, \( \eta^E \omega = \int_{\omega_{\min}}^{\omega^C} \omega f(\omega) \, d\omega \), while the conditional expectation of \( \omega \) when entrepreneur’s project is insolvent by, \( (1 - \eta^E) \omega = \int_{\omega_{\min}}^{\omega^C} \omega f(\omega) \, d\omega \). The share of profitable projects is computed as, \( \eta^E = \Pr[\omega \geq \omega^C] = \int_{\omega_{\min}}^{\infty} f(\omega) \, d\omega = (\omega_{\min}/\omega^C)^{\kappa} \). The conditional expectations is computed via, \( \bar{\omega} = E[\omega|\omega \geq \omega^C] = \int_{\omega_{\min}}^{\omega^C} \omega f(\omega) \, d\omega / \int_{\omega_{\min}}^{\infty} f(\omega) \, d\omega = \frac{\kappa}{\kappa - 1} \omega^C \). Since \( E[\omega] = \eta^E E[\omega|\omega \geq \omega^C] + (1 - \eta^E) E[\omega|\omega < \omega^C] = 1 \), then \( \omega = \frac{(1 - \eta^E) \omega}{(1 - \eta^E)} \).

The aggregation of financial projects \( \omega, \int_{Q_{i,t+1}}^{+\infty} \omega P_{i,t+1}^k Q_{i,t+1} K_{i,t+1} (e, \omega) \, dF(\omega), \) leads to the following expression of the expected profitability,

\[
\Pi^E_{t,t+1} (e) = \mathbb{E}_t \left\{ \frac{\tilde{\omega}_{i,t+1} P_{i,t+1}^k Q_{i,t+1} K_{i,t+1} (e) - P_{i,t}^L (e) L_{i,t+1}^H (e)}{1 - \eta^E_{i,t+1}} \right\}
\]

(A.15)

### A.5.3 Profit Maximization and the Financial Accelerator

To introduce a financial accelerator mechanism, we borrow a concept of De Grauwe (2010) applied in a different context, by assuming that entrepreneurs’ forecasts regarding the aggregate profitability of a given project \( \tilde{\omega}_{i,t} (e) \) are optimistic (i.e., biased upwards). The perceived \textit{ex ante} value of profitable projects is defined by the isoleastic function,

\[
g \left( \tilde{\omega}_{i,t+1}, \omega^Q_{i,t} \right) = \gamma_i \left( \tilde{\omega}_{i,t+1} \right)^{\frac{x_{i,t+1}}{\gamma_i \left( \tilde{\omega}_{i,t+1} \right)}}.
\]

\( ^{13} \)Assuming optimistic firms is motivated empirically, Bachmann and Elstner (2013) find evidence of such expectations for German firms using microdata. This hypothesis of the expectations of the private sector is very close to the utility functions introduced by Goodhart et al. (2005) for bankers. In our setting, the financial accelerator does not result from a moral hazard problem but rather from a bias in the expectations of the private sector.
where $\kappa_i$ is the elasticity of the external finance premium and $\gamma_i$ is a scale parameter. Thus, ex-ante the entrepreneur chooses a capital value of $K_{i,t+1}(e)$ that maximizes its expected profit defined as,

$$\max_{\{K_{i,t+1}(e)\}} \mathbb{E}_t \left\{ \eta_i^E_{t+1} \left[ g(\tilde{\omega}_{i,t+1}) R_{i,t+1}^k Q_{i,t+1}(e) - P_{i,t}^L(e) L_{i,t+1}^H(e) \right] \right\}. \tag{A.16}$$

Using the characteristics of the Pareto distribution, the expected spread required by representative entrepreneur $e$ to undertake the decision to finance firms’ investment is,

$$S_{i,t}(e) = \frac{\mathbb{E}_t R_{i,t+1}^k}{P_{i,t}^L(e)} = \gamma_i^{\kappa_i-1} \left[ \frac{\kappa}{\kappa - 1} \left( 1 - \frac{N_{i,t+1}(e)}{Q_{i,t} K_{i,t+1}(e)} \right) \right]^\kappa_i. \tag{A.17}$$

The size of the accelerator is determined by the elasticity of the external finance premium $\kappa_i$. For $\kappa_i > 0$, the external finance premium is a positive function of the leverage ratio, $Q_{i,t} K_{i,t+1}(e) / N_{i,t+1}(e)$, so that an increase in net wealth induces a reduction of the external finance premium. This phenomenon disappears if $\kappa_i = 0$. Furthermore, a shock that hits the entrepreneur net wealth $N_{i,t+1}(e)$ will also affect the rentability of the physical capital in the economy. As the rentability of capital is a cost for the intermediate sector, a variation in the net wealth will have aggregate consequences on goods supply through the channel of the capital market as underlined by Gilchrist et al. (2009b). The amount of capital of non-profitable entrepreneurs’ investment projects is consumed in terms of home final goods $P_{i,t} (1 - \eta_{i,t}^E) \tilde{\omega}_{i,t}(e) R_{i,t}^k Q_{i,t-1} K_{i,t}(e)$. Thus the net wealth of the entrepreneur in the next period is equal to,

$$N_{i,t+1}(e) = \left( 1 - \tau_i^E \right) \frac{\Pi^E_{i,t}(e)}{\epsilon_i^{N_i}}, \tag{A.18}$$

where $\epsilon_i^{N_i}$ is an exogenous process of net wealth destruction and $\tau_i^E$ is a proportional tax on the profits of the bank.

### A.6 Capital Goods Producers

The capital supplier is an alternative decentralization scheme in which a new type of firms, capital producers, make the capital supply and utilization decisions.

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14 The elasticity of the external finance premium expresses the degree of bias in estimating the expected rentability of entrepreneurs’ projects such that if $\tilde{\omega} > 1$ and $\kappa_i > 0$ then $g(\tilde{\omega}) > \tilde{\omega}$. Expressed à la De Grauwe (2010), $\mathbb{E}_t^{opt} \tilde{\omega}_{i,t+1} = \mathbb{E}_t \gamma_i (\tilde{\omega}_{i,t+1})^{\kappa_i/(\kappa_i - 1)}$ where $\mathbb{E}_t^{opt}$ is the expectation operator of optimistic entrepreneurs.

15 This parameter is needed to make the steady state independent of $\kappa_i$, such that $\gamma_i = \omega_i^{1/(1-\kappa_i)}$. 

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A.6.1 Capital Supply Decisions

The suppliers of capitals lend capital to the intermediate firms, once it is financed by the entrepreneurs. Capital suppliers are homogeneous and distributed over a continuum normalized to one. The representative capital supplier \( k \in [0; 1] \) acts competitively to supply a quantity of capital \( K_{i,t+1}(k) \) to intermediate firms and invest a quantity of final goods \( I_{i,t}(k) \) to keep it productive. We assume that it is costly to invest, i.e. it has to pay an adjustment cost on investment,

\[
AC_{i,t}^I(k) = \chi^I_i \left( \frac{e^{I_{i,t}} I_{i,t}(k)}{I_{i,t-1}(k)} - 1 \right)^2 \tag{A.19}
\]

where \( \chi_i^I \) is the adjustment cost. Thus the capital stock of the representative capital supplier evolves according to,

\[
K_{i,t+1}(k) = (1 - AC_{i,t}^I(k)) I_{i,t}(k) + (1 - \delta) K_{i,t}(k) \tag{A.20}
\]

The capital producer produces the new capital stock \( Q_{i,t} K_{i,t+1}(k) \) by buying the deprecated capital and investment goods. The project of the representative supplier thus writes,

\[
\Pi^k_{i,t}(k) = Q_{i,t} K_{i,t+1}(k) - (1 - \delta) Q_{i,t} K_{i,t}(k) - P_{i,t}^I I_{i,t}(k), \tag{A.21}
\]

Replacing (A.20) in (A.21), the representative capital supplier chooses \( I_{i,t}(k) \) to maximize profits,

\[
\max_{\{I_{i,t}(k)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \frac{\chi_i^c}{\chi_i^c} \Pi_{i,t+\tau}^k(k) \right\},
\]

where \( \beta^\tau \chi_i^c \) is the household subjective discount factor. The price of capital renting thus solves,

\[
Q_{i,t} = P_{i,t}^I + Q_{i,t} \partial (I_{i,t}(k) AC_{i,t}^I(k)) \frac{\partial I_{i,t}(k)}{\partial I_{i,t}(k)} + \beta \mathbb{E}_t \frac{\chi_i^c}{\chi_i^c} Q_{i,t+1} \frac{\partial (I_{i,t+1}(k) AC_{i,t+1}^I(k))}{\partial I_{i,t}(k)}.
\tag{A.22}
\]

Ignoring investment adjustment costs in this last expression (i.e. imposing \( \chi_i^{I} = 0 \)), we simply get, \( Q_{i,t} = P_{i,t}^I \). \( Q_{i,t} \) stands for the asset price given the adjustment costs on investment production function. The derivatives of \( AC_{i,t}^I \) in \( I_{i,t} \) are,

\[
\frac{\partial [I_{i,t} AC_{i,t}^I]}{\partial I_{i,t}} = \frac{\chi_i^I}{2} \left( 3 \left( \frac{e^{I_{i,t}} I_{i,t}}{I_{i,t-1}} \right)^2 + 1 - 4 \frac{e^{I_{i,t}} I_{i,t}}{I_{i,t-1}} \right)
\]

\[
\frac{\partial [I_{i,t+1} AC_{i,t+1}^I]}{\partial I_{i,t}} = \chi_i^I \left( \frac{e^{I_{i,t+1}} I_{i,t+1}}{I_{i,t+1}} \right)^2 - \left( \frac{e^{I_{i,t+1}} I_{i,t+1}}{I_{i,t+1}} \right)^3
\]
A.6.2 The Rentability of one Unit of Capital

Equation (A.23) takes into account the assumption that households make the capital accumulation. The capital supplier is actually an alternative decentralization scheme in which firms make the capital supply decisions. To get the equation of the \textit{ex post} rentability of one unit of capital, we must suppose that the household supplies capital by investing \(P_{i,t}^I I_{i,t} (j)\) and earning revenues from renting capital \(Z_{i,t} K_{i,t} (j)\). Maximizing utility under capital accumulation constraint (A.20), the FOCs write,

\[
(\partial B_{i,t+1} (j)) : \mathbb{E}_t \left\{ \frac{e^{\epsilon_{u,t}^c} \lambda_{i,t}^c}{\sigma_{i,t+1}^c} \frac{P_{i,t+1}^c}{P_{i,t}^c} \right\} = \beta \left( 1 + R_{i,t}^D \right) \\
(\partial K_{i,t+1} (j)) : \lambda_{i,t}^k - \beta \mathbb{E}_t \lambda_{i,t+1}^c Z_{i,t+1} - \beta (1 - \delta) \mathbb{E}_t \lambda_{i,t+1}^c
\]

where \(\lambda_{i,t}^c (\lambda_{i,t}^k)\) is the Lagrangian multiplier on the budget constraint (capital accumulation constraint). Tobin’s Q is defined by \(Q_t = \lambda_{i,t}^k / \lambda_{i,t}^c\), then replacing in the FOC (\(\partial K_{i,t+1} (j)\)),

\[
Q_{i,t} = \beta \frac{\lambda_{i,t+1}^c}{\lambda_{i,t}^c} [Z_{i,t+1} + Q_{i,t+1} (1 - \delta)]
\]

Combining the previous equation with FOC (\(\partial D_{i,t+1}^d (j)\)) leads to the no arbitrage condition between deposits and capital assets,

\[
\frac{1 + R_{i,t}^D}{1 + AC_{i,t}^{DP}} = \mathbb{E}_t \left[ Z_{i,t+1} + Q_{i,t+1} (1 - \delta) \right] \frac{e^{\epsilon_{u,t}^c}}{\sigma_{i,t+1}^c}
\]

Following Bernanke et al. (1999) we lag the previous equation and replace \(R_{i,t}^D\) by \(R_{i,t+1}^k\) to get the \textit{ex post return} of capital. Since capital supply decisions are decentralized, we suppose that they are independent of preference shocks as in Smets Wouters (2003, 2007), we apply the same hypothesis for adjustment costs on deposits. Finally, the return of holding one unit of capital from \(t - 1\) to \(t\) is determined by,

\[
\mathbb{E}_{t-1} (1 + R_{i,t}^k) = \frac{Z_{i,t} + (1 - \delta) Q_{i,t}}{Q_{i,t-1}} \quad (A.23)
\]

A.7 The Banking Sector

This sector is made up of two distinct branches: a continuum of monopolistic banks and a financial intermediary. Monopolistic banks \(b \in [0; 1]\) provide different types of loans and deposit services and set interest rate in a Calvo basis. The financial intermediary is a CES packer, he produces one homogenous loan and deposit service using the different varieties from banks. Banks may engage in cross-border loans but deposit system remain closed.
A.7.1 The Financial Intermediary (CES Packer)

The financial intermediary has deposit and loan activities, it acts as a loan and deposit bundler in a perfectly competitive market. Wholesale branches supply differentiated types of deposits and loans packed by retail branches. It maximizes profits subject to the supply curve,

\[
\max_{D_{i,t+1}(b), L_{i,t}(b)} \quad R^{D}_{i,t} D_{i,t+1}^d + R^{L}_{i,t} L_{i,t+1}^d + \mathcal{G} \left( R^{D}_{i,t} (b) D_{i,t+1}(b) \right) + \mathcal{G} \left( R^{L}_{i,t} (b) L_{i,t+1}(b) \right)
\]

s.t. \( D_{i,t+1} = \left( \frac{1}{n_i} \frac{n_i}{D_{i,t}} \mathcal{G} \left( D_{i,t+1}(b) (\frac{D_{i,t}-1}{D_{i,t}}) \right) \right)^{\frac{1}{\epsilon_D}} \)

s.t. \( L_{i,t+1} = \left( \frac{1}{n_i} \frac{n_i}{L_{i,t}} \mathcal{G} \left( L_{i,t+1}(b) (\frac{L_{i,t}-1}{L_{i,t}}) \right) \right)^{\frac{1}{\epsilon_L}} \)

Where \( L_{i,t+1}^d \) is the loans demand from home and foreign entrepreneurs and \( \mathcal{G}(.) \) is the aggregator function. Deposits and loans are imperfect substitute with elasticity of substitution \( \epsilon_D < -1 \) and \( \epsilon_L > 1 \). We find the intermediate demand functions associated from the previous problem are,

\[
D_{i,t+1}(b) = \frac{1}{n_i} \left( \frac{R^{D}_{i,t}(b)}{R^{D}_{i,t}} \right)^{-\epsilon_D} D_{i,t+1}, \quad \forall b \quad (A.24)
\]

\[
L_{i,t+1}^d(b) = \frac{1}{n_i} \left( \frac{R^{L}_{i,t}(b)}{R^{L}_{i,t}} \right)^{-\epsilon_L} L_{i,t+1}, \quad \forall b \quad (A.25)
\]

Thus the aggregate price index of all varieties in the economy writes,

\[
R^{D}_{i,t} = \left[ \frac{1}{n_i} \mathcal{G} \left( R^{D}_{i,t} (b)^{1-\epsilon_D} \right) \right]^{\frac{1}{1-\epsilon_D}} \quad (A.26)
\]

\[
R^{L}_{i,t} = \left[ \frac{1}{n_i} \mathcal{G} \left( R^{L}_{i,t} (b)^{1-\epsilon_L} \right) \right]^{\frac{1}{1-\epsilon_L}} \quad (A.27)
\]

A.7.2 The Commercial Bank

In each country, the banking sector finances investment projects to home and foreign entrepreneurs by supplying one-period loans. The banking system is heterogeneous with regard to liquidity, and banks engage in interbank lending at the national and international levels. Thus, cross-border loans are made of corporate loans (between banks and entrepreneurs) and interbank loans.

To introduce an interbank market, we suppose that the banking system combines liquid and illiquid banks. We assume that a share of banks \( \lambda \) are illiquid (i.e.
credit constrained), while the remaining banks share $1 - \lambda$ are liquid and supply loans to entrepreneurs and to illiquid banks. We assume that a liquid bank is characterized by her direct accessibility to the ECB fundings. Conversely, an illiquid bank does not have access to the ECB fundings. This assumption is empirically motivated: in the Eurosystem, only a fraction of the 2500 banks participates regularly to the bidding process in main refinancing operations of the ECB while the others rely on interbank funding, as underlined by Gray et al. (2008). Extending this assumption in an international perspective, illiquid banks can borrow from both domestic and foreign liquid banks, which gives rise to cross-border interbank lending flows.

**Illiquid Banks** The representative share $\lambda$ of illiquid bank $b$ in country $i$ operates under monopolistic competition to provide a quantity of loans $L^s_{i,t+1}(b)$ to entrepreneurs that is financed by deposits $D_{i,t+1}(b)$ from households, interbank loans $IB_{i,t+1}(b)$ from the interbank market (with a one-period maturity) at a rate $P^I_{i,t}$. The balance sheet of the bank writes,

$$L^s_{i,t+1}(b) = IB^H_{i,t+1}(b) + BK_{i,t+1}(b) + e_{i,t}(b), \quad (A.28)$$

where $L^s_{i,t+1}(b)$ is the loan supply of borrowing banks, $IB^H_{i,t+1}(b)$ is the interbank loans supplied by liquid banks subject to external habits, $BK_{i,t+1}(b)$ is the bank capital and $e_{i,t}(b)$ are other liabilities in the balance sheet of the bank that are not considered in the model:\footnote{We suppose that they follow an exogenous $AR(1)$ shock process $\varepsilon^B_{i,t}$ such that, $e_{i,t} = e^B_{i,t} \tau_{i,t}$, this shock captures some aggregate movements in the funding constraint of banks.} to close the model, we assume that the cost of these liabilities is decided by the central bank. We suppose that the demand for interbank funds are subject to external habits at a degree $h^B_i$ where $IB^H_{i,t+1}(b) = IB^d_{i,t+1}(b) - h^B_i(IB^d_{i,t+1} - T^B_i)$. These habits are deemed necessary to catch-up the autocorrelation observed in the supply of loans:\footnote{In the fit exercise, DSGE models with banking are estimated on the outstanding amount of loans contracted in the economy. Since DSGE models only include one-period maturity loans, external habits are a tractable way to catch up the high persistence in the loan contracts without modifying the steady state. Guerrieri et al. (2012) develops a similar financial friction in the borrowing constraint of entrepreneurs.}

This bank engages in international corporate loans. In this setting, we assume that there is no discrimination between borrowers, so that the representative and risk-neutral bank serves both domestic and foreign entrepreneurs without taking into account specificities regarding the national viability of projects. Under this assumption, bank default expectation regarding entrepreneurs’ projects is defined
by a geometric average,

$$
\eta_{i,t} = \frac{E_{i,t} E_{j,t}}{\bar{\eta}} \eta_{i,t} \eta_{j,t}^{\alpha_i}, \quad \forall i \neq j \in \{c, p\}
$$

(A.29)

where $\eta_{i,t+1}^E$ is the default rate in country $i \in \{c, p\}$ of entrepreneurs and $(1 - \alpha_i^L)$ measures the home bias in corporate loan distribution and $\bar{\eta}$ is the steady state level share of profitable projects\(^{18}\). The profits of the representative illiquid bank are,

$$
E_{t+1} \Pi_{i,t+1}^{ill} (b) = \left[ \mathbb{E}_t \eta_{i,t+1} + (1 - \mu^b) (1 - \eta_{i,t+1}) \right] (1 + R_{i,t}^d (b)) L_{s,t}^i (b) - (1 + R_{i,t}^d (b)) D_{i,t} \Delta_t 3}\mathbb{E}_t
$$

Revenues from loan supply activities

$$
- (1 + P_{i,t}^B I B_{i,t+1}^d (b)) - (1 + R_t) I i a b_{i,t} (b) \quad F_i \left( \frac{L_{i,t+1}^s (b)}{B K_{i,t+1}^{ill} (b)} \right) B K_{i,t+1}^{ill} (b)
$$

Deposit cost

Interbank cost

exogenous funds cost

Basel I capital requirements

where $\mu^b \in [0; 1]$ denotes the loss-given-default, i.e. the percentage of the amount owed on a defaulted loan that the bank is not able to recover and $F_i (\cdot)$ denotes the basel I-like capital requirement penalty function paid in terms of bank capital and reads as in Gerali et al. (2010),

$$
F_i (x_t) = 0.5 \chi_i^K (x_t - \bar{x})^2
$$

where $\chi_i^K$ is the estimated size of the penalty cost function. Thus, the marginal cost of one unit of loan $MC_{i,t}^{ill} (b)$ by the illiquid bank is the solution of the expected profit $E_t \Pi_{i,t+1}^{ill} (b)$ optimization problem,

$$
\max_{L_{i,t+1}^s (b), I B_{i,t+1}^d (b)} E_t \Pi_{i,t+1}^{ill} (b)
$$

(A.31)

We assume that rates are sticky as Darracq-Pariès et al. (2011), banks must solve a two-stage problem. In the first stage, banks choose the optimal supply of credit and deposit services in a perfectly competitive input markets. The marginal cost of one unit of loan, denoted $MC_{i,t}^{ill} (b)$, is the same across illiquid banks and writes,

$$
1 + MC_{i,t}^{ill} (b) = 1 + MC_{i,t}^{ill} = 1 + P_{i,t}^B + F^I_{i,t} \frac{E_t \eta_{i,t+1}}{\mathbb{E}_t \eta_{i,t+1}},
$$

(A.32)

so that each bank decides the size of the spread depending on the expected failure rate of its customers $E_t \eta_{i,t+1}$, the interbank rate in the economy and the penalty

\(^{18}\)We divide respectively $\eta_{i,t}^E$ and $\eta_{j,t}^E$ by $\bar{\eta}$ to have a symmetric steady state such that $\eta_i^E = \bar{\eta} = \bar{\eta}_p$, this hypothesis is neutral on the log-linear version of the model.
costs when bank capital to asset ratio deviate from its steady state\textsuperscript{19}. The bank has access to domestic and foreign interbank loans to meet its balance sheet. The total amount borrowed by the representative bank writes,

\[
IB^d_{i,t+1} (b) = \left( (1 - \alpha_i^B) \right)^{1/\xi} IB^d_{h,i,t+1} (b) + \left( \alpha_i^B \right)^{1/\xi} IB^d_{f,i,t+1} (b),
\]

where parameter $\xi$ is the elasticity of substitution between domestic and foreign interbank funds, $\alpha_i^B$ represents the percentage of cross-border interbank loan flows in the monetary union and $IB^d_{h,i,t+1} (b)$ (resp. $IB^d_{f,i,t+1} (b)$) the amount of domestic (resp. foreign) loans demanded by borrowing bank $b$ in country $i$. The total cost incurred by illiquid banks to finance interbank loans, $1 + P^IB_{i,t}$, is thus defined according to the CES aggregator,

\[
1 + P^IB_{i,t} = \left( (1 - \alpha^B_i) \right) \left( 1 + R^IB_{h,t} \right)^{1-\xi} + \alpha_i^B \left( 1 + R^IB_{f,t} \right)^{1-\xi} \right)^{1/(1-\xi)}, \tag{A.34}
\]

where $1 + R^IB_{h,t}$ (resp. $1 + R^IB_{f,t}$) is the cost of loans obtained from home (resp. foreign) banks in country $i$. The decision to borrow from a particular bank is undertaken on the basis of relative interbank national interest rates,

\[
IB^d_{h,i,t+1} (b) = \left( 1 - \alpha_i^B \right) \left[ \frac{1 + R^IB_{h,t}}{1 + P^IB_{i,t}} \right]^{-\xi} IB^d_{i,t+1} (b), \quad \text{and} \quad IB^d_{f,i,t+1} (b) = \alpha_i^B \left[ \frac{1 + R^IB_{f,t}}{1 + P^IB_{i,t}} \right]^{-\xi} IB^d_{i,t+1} (b).
\]

Here, cross-border lending is measured through the values undertaken by $IB^d_{h,i,t+1} (b)$, (i.e., interbank loans contracted by liquid foreign banks from domestic overliquid banks). Finally following Hirakata et al. (2009), the bank capital accumulation process of illiquid banks ($BK_{ill,t+1}^i (b)$) is determined by,

\[
BK_{ill,t+1}^i (b) = \left( 1 - \tau_{ill}^i \right) \Pi_{ill,t}^i (b), \tag{A.35}
\]

where $\tau_{ill}^i$ is a proportional tax on the profits of the bank.

**Liquid Banks** The representative share of $1 - \lambda$ liquid bank $b$ in country $i$ operates under monopolistic competition to provide a quantity of loans $L^s_{i,t+1} (b)$ to entrepreneurs. It also provides a quantity of interbank loans $IB^s_{i,t+1} (b)$ to illiquid banks. We suppose that the intermediation process between liquid and illiquid banks is costly: we introduce a convex monitoring technology à la Cúrdia and Woodford (2010) and Dib (2010) with a functional form $AC^IB_{i,t+1}^j (b) = \frac{\chi^IB}{2} \left( IB^s_{i,t+1} (b) - \overline{IB}^s_i (b) \right)^2$ where parameter $\chi^IB_i$ is the level of financial frictions\textsuperscript{19} $F_{i,t}$ denotes the derivate in $L^s_{i,t+1} (b)$ of the capital requirement cost function $F_i (\cdot)$.

\[19\]
between liquid banks in country $i$ and home and foreign illiquid banks$^{20}$. Loans created by the liquid bank are financed by one-period maturity loans from the central bank ($L^{\text{ECB}}_{i,t+1}(b)$) at the refinancing interest rate $R_t$. Finally, the bank’s balance sheet is defined by,

$$ L^s_{i,t+1}(b) + IB^s_{i,t+1}(b) = L^{\text{ECB}}_{i,t+1}(b) + BK_{i,t+1}(b) + D_{i,t} + liab_{i,t}(b). $$

According to the behavior of illiquid banks, we assume that there is no discrimination between borrowers. The one-period profit of the bank is determined by,

$$ liq_{i,t}(b) = (1 - \mu^b \mathbb{E}_t \eta_{i,t+1}) \left(1 + R^L_{i,t}(b)\right) L^s_{i,t+1}(b) + (1 + R^B_{i,t}(b)) IB^s_{i,t+1}(b) 
- (1 + R_t) L^{\text{ECB}}_{i,t+1}(b) - AC^B_{i,t+1}(b) - F_i \left(\frac{L^s_{i,t+1}(b) + IB^s_{i,t+1}(b)}{BK_{i,t+1}(b)}\right) BK^{liq}_{i,t+1}(b). $$

The representative bank solves he profit maximization problem,

$$ \max_{L^s_{i,t+1}(b), IB^s_{i,t+1}(b), L^{\text{ECB}}_{i,t+1}(b)} \Pi^{liq}_{i,t}(b) \quad (A.36) $$

under the balance sheet constraint. The marginal cost of one unit of loan $MC^{liq}_{i,t}(b)$ that solves the maximization problem is,

$$ 1 + MC^{liq}_{i,t}(b) = 1 + MC^s_{i,t} = \frac{1 + R_t + F^L_{i,t}}{\mathbb{E}_t \eta_{i,t+1}}. \quad (A.37) $$

Similarly to the illiquid bank, bank capital evolves according to Equation (A.35) such that $BK^{liq}_{i,t+1}(b) = (1 - \tau^{liq}_i) \Pi^{liq}_{i,t}(b)$.

There are two interest rates to be determined: the interest rate on the interbank market and the interest rate on corporate loans. First, on a perfectly competitive market, the interbank rate in country $i$ is determined from the problem (A.36),

$$ 1 + R^B_{i,t}(b) = \chi^B_i \left(IB^s_{i,t+1}(b) - \overline{BB}_i^s(b)\right) + (1 + R_t) + F^B_{i,t}, \quad (A.38) $$

where, $\chi^B_i$ is a cost parameter, $IB^s_{i,t+1}(b)$ is the amount of interbank loans contracted in period $t$ with a one period maturity and $\overline{BB}_i^s(b)$ is the steady state value of interbank loans.

$^{20}$Contrary to Cúrdia and Woodford (2010) but in the same vein of Dib (2010), the monitoring technology does not alter the steady state of the model to keep the estimation of $\chi^B_i$ as simple as possible. Several papers refer to monitoring technology functions in the intermediation process of banks, see for example Goodfriend and McCallum (2007) or Casares and Poutineau (2011).
The loan supply decisions  In the second stage problem solved by banks, the interest rate charged by banks of country $i$ on corporate loans accounts for the liquidity of the national banking system. Anticipating over symmetric issues at the equilibrium to improve the tractability of the model, we assume that all banks belonging to a national banking system share the same marginal cost of production, reflecting the average liquidity degree of national banks. Aggregate marginal cost $MC_{i,t}^L$ combines outputs from liquid and illiquid banks of country $i$ according to\(^{21}\),

$$1 + MC_{i,t}^L = \left(1 + MC_{i,t}^{ill} \right) \lambda \left(1 + MC_{i,t}^{ill} \right)^{(1-\lambda)}.$$ \hfill (A.39)

Under Calvo pricing, banks operates under monopolistic competition and sets the interest rate on loans contracted by entrepreneurs on a staggered basis as in Darracq-Pariès et al. (2011). A fraction $\theta^L_{t}$ of banks is not allowed to optimally set the credit rate such that, $R^L_{i,t} (b) = R^L_{i,t-1} (b)$. Assuming that it is able to modify its loan interest rate with a constant probability $1 - \theta^L_{t}$, it chooses $R^L_{i,t} (b)$ to maximize its expected sum of profits $\Pi^L_{i,t} (b)$,

$$\max_{\{R^L_{i,t} (b)\}} E_t \left\{ \sum_{\tau=0}^{\infty} \left( \theta^L_{t} \right)^{\tau} \frac{\lambda^C_{i,t+\tau}}{\lambda^c_{i,t}} \left[ R^L_{i,t} (b) - MC^L_{i,t+\tau} \right] L^s_{i,t+1+\tau} (b) \right\},$$ \hfill (A.40)

subject to the demand constraint from retail banks,

$$L^s_{i,t+1+\tau} (b) = \left( \frac{R^L_{i,t} (b)}{R^L_{i,t+\tau}} \right)^{-\epsilon_L} L^d_{i,t+1+\tau}, \quad \tau > 0,$$ \hfill (A.41)

where $L^s_{i,t+1}$ represents the quantity of the loans produced in country $i$, $\lambda^c_{i,t}$ is the household marginal utility of consumption.

Finally, the first order condition writes,

$$\sum_{\tau=0}^{\infty} \left( \theta^L_{t} \right)^{\tau} \frac{\lambda^C_{i,t+\tau}}{\lambda^c_{i,t}} \left[ R^L_{i,t} (b) - \frac{\epsilon_L}{\epsilon_L - 1} e^{\epsilon_L t+\tau} MC^L_{i,t+\tau} \right] L^s_{i,t+1+\tau} (b) = 0$$ \hfill (A.42)

The deposit decisions  When taking their deposit supply decisions, banks also solves a two-stages problem due to imperfect price adjustments on this market. The deposit market is very special since households supply deposits to banks, but banks decides of the remuneration of deposits. Following Gerali et al. (2010) and Darracq-Pariès et al. (2011), we suppose that the marginal cost of one unit of deposit is determined by the central bank,

$$MC^D_{i,t} (b) = MC^D_{i,t} = R_t$$ \hfill (A.43)

\(^{21}\)We borrow this aggregation procedure from the solution introduced by Gerali et al. (2010), to aggregate borrowing and saving households labor supply.
so that each bank decides the size of the spread depending on the expected failure rate of its customers.

Under Calvo pricing, banks set the interest rate on deposits supplied by households on a staggered basis as in Darracq-Pariès et al. (2011). A fraction \( \theta_i^D \) of banks is not allowed to optimally set the credit rate such that, \( R_{i,t}^D (b) = R_{i,t-1}^D (b) \). Assuming that it is able to modify its loan interest rate with a constant probability \( 1 - \theta_i^D \), it chooses \( R_{i,t}^D (b) \) to maximize its expected sum of profits \( \Pi_{i,t}^D (b) \),

\[
\max_{\{R_{i,t}^D (b)\}} E_t \left\{ \sum_{\tau=0}^{\infty} \left( \theta_i^D \beta \right)^\tau \frac{\lambda_{i,t+\tau}}{\lambda_{i,t}^C} \left[ MC_{i,t+\tau}^D - R_{i,t}^D (b) \right] D_{i,t+1+\tau} (b) \right\}, \tag{A.44}
\]

subject to the demand constraint from retail banks,

\[
D_{i,t+1+\tau} (b) = \left( \frac{R_{i,t}^D (b)}{R_{i,t+\tau}^D} \right)^{-e_D} D_{i,t+1+\tau}^d, \quad \tau > 0, \tag{A.45}
\]

where \( L_{i,t+1}^s \) represents the quantity of the loans produced in country \( i \), \( \lambda_{i,t}^C \) is the household marginal utility of consumption.

Finally, the first order condition writes,

\[
\sum_{\tau=0}^{\infty} \left( \theta_i^D \beta \right)^\tau \frac{\lambda_{i,t+\tau}}{\lambda_{i,t}^C} \left[ R_{i,t}^D (b) - \frac{e_D}{\epsilon_D - 1} e^{e_D MC_{i,t+\tau}^D} \right] D_{i,t+1+\tau} (b) = 0 \tag{A.46}
\]

where \( e^{e_D} \) is an ad hoc time-varying mark-up shock to the deposit rate equation.

### A.8 Authorities

National governments finance public spending by charging a proportional taxes on the net wealth of banks \( \tau^b \), entrepreneurs \( \tau^E \) and by receiving a total value of taxes \( G (T_{i,t} (j)) \) from households. The budget constraint of the national government writes,

\[
G (T_{i,t} (j)) + \tau^E G \left( P_{i,t}^c N_{i,t} (e) \right) + \tau^b G \left( P_{i,t}^c BK_{i,t} (b) \right) = P_{i,t} G_{i,t} = P_{i,t} G \left( G_{i,t} (i) \right)^{\epsilon_D - 1} = P_{i,t} G_{i,t}^G \tag{A.47}
\]

where \( G_{i,t} \) is the total amount of public spending in the \( i \)th economy that follows and AR(1) shock process. The government demand for home goods writes, \( G_{i,t} (i) = \left( \frac{P_{i,t} (i)}{P_{i,t}} \right)^{\epsilon_D} G_{i,t} \).

Concerning federal monetary policy, the general expression of the interest rule implemented by the monetary union central bank writes,

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^\rho \left( \frac{\pi_C^u}{\pi_{u,t}} \phi^\rho \left( \frac{Y_{u,t}}{Y_{u,t-1}} \right) \phi^{\Delta u} \right)^{(1-\rho)^\rho} e^{e_i^R} \tag{A.47}
\]

51
where \( \varepsilon^R_t \) is a monetary policy shock common to the monetary union members, \( \phi^* \) is the inflation target parameter, \( \phi^\Delta_y \) is the GDP growth target. Recall that 
\[
\pi^c_{u,t} = \left( \pi^C_{c,t} \right)^n \left( \pi^C_{p,t} \right)^{1-n}
\] and 
\[
Y_{u,t} = \left( Y_{c,t} \right)^n \left( Y_{p,t} \right)^{1-n}.
\]

### A.9 Aggregation and Market Equilibrium

After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) substituting the relevant demand functions, (iv) normalizing the total size of the monetary union \((n_c + n_p = 1)\) such that the size of the core area is \(n\) and the peripheral area size is \(1 - n\), we can deduct the general equilibrium conditions of the model. Now we can express the aggregation function of variable \(X_t(x)\) as,

\[
G(X_{i,t}) = \begin{cases} 
\int_0^n X_{i,t}(x) \, dx & \text{for } i = c \\
\int_n^1 X_{i,t}(x) \, dx & \text{for } i = p
\end{cases}
\]  

(A.48)

#### A.9.1 Goods Market

From equation ((A.8)), the aggregate price index of the national goods evolves according to,

\[
P_{i,t}^{1-\varepsilon_p} = \theta^P_i \left[ P_{i,t-1} \left( \frac{P_{i,t-1}}{P_{i,t-2}} \right)^{\varepsilon_p} \right]^{1-\varepsilon_p} + (1 - \theta^P_i) \left( P^*_{i,t} \right)^{1-\varepsilon_p} \]  

(A.49)

The equilibrium condition on the final goods market writes is defined by the aggregation of equation ((A.7)),

\[
G(Y_{i,t}(i)) = Y_{i,t}^d G \left( \frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\varepsilon_p}
\]

where \( G(Y_{i,t}(i)) = \varepsilon^R_{i,t} G \left( K_{i,t} (i)^\alpha H^d_{i,t} (i)^{1-\alpha} \right) \) is the aggregation of intermediate goods suppliers and \( Y_{i,t}^d \) is the resources constraint,

\[
Y_{i,t}^d = C_{h,i,t} + C_{f,i,t} + (1 + AC^L_{h,i,t}) I_{h,i,t} + (1 + AC^L_{f,i,t}) I_{f,i,t} + G_{i,t} + AC^D_{i,t}
\]
Thus, replacing the demand functions of foreign and home goods (consumption and investment), we finally obtain the home final goods market equilibrium,

\[
\frac{Y_{c,t}}{\Delta^P c_{t}} = (1 - \alpha_c^C) \left( \frac{P_{c,t}}{P_{c,t}^P} \right)^{-\mu} C_{c,t} + (1 - \alpha_c^I) \left( \frac{P_{c,t}}{P_{c,t}^I} \right)^{-\mu} (1 + AC_{c,t}^I) I_{c,t} \tag{A.50}
\]

\[
+ \frac{n - 1}{n} \left( \alpha_c^C \left( \frac{P_{c,t}}{P_{c,t}^P} \right)^{-\mu} C_{p,t} + \alpha_c^I \left( \frac{P_{c,t}}{P_{c,t}^I} \right)^{-\mu} (1 + AC_{c,t}^I) I_{p,t} \right) \tag{A.51}
\]

\[
+ G_{c,t} + AC_{c,t}^D
\]

where \( \Delta^P c_{t} = \mathcal{G} \left( \frac{P_{c,t}(i)}{P_{c,t}} \right)^{-\epsilon_P} \) denotes the price dispersion term, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. Since we perform a first approximation of the model, the price dispersion terms disappear. For the foreign economy,

\[
\frac{Y_{p,t}}{\Delta^P p_{t}} = (1 - \alpha_p^C) \left( \frac{P_{p,t}}{P_{p,t}^P} \right)^{-\mu} C_{p,t} + (1 - \alpha_p^I) \left( \frac{P_{p,t}}{P_{p,t}^I} \right)^{-\mu} (1 + AC_{p,t}^I) I_{p,t} \tag{A.52}
\]

\[
+ \frac{n - 1}{n} \left( \alpha_c^C \left( \frac{P_{p,t}}{P_{p,t}^P} \right)^{-\mu} C_{c,t} + \alpha_c^I \left( \frac{P_{p,t}}{P_{p,t}^I} \right)^{-\mu} (1 + AC_{c,t}^I) I_{c,t} \right) \tag{A.53}
\]

\[
+ G_{p,t} + AC_{p,t}^D
\]

To close the model, adjustment costs on deposits are entirely home biased \( AC_{i,t}^D = \mathcal{G} \left( AC_{i,t}^D (i)^{(\epsilon_P - 1)/\epsilon_P} \right)^{\epsilon_P/(\epsilon_P - 1)} \), the associated demand function writes, \( AC_{i,t}^D (i) = \left( \frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\epsilon_P} AC_{i,t}^D \).

**A.9.2 Loan Market**

The equilibrium on loan market is defined by the aggregate demand function from loans packers (??),

\[
\mathcal{G} \left( L_{i,t+1}^L (b) \right) = \Delta^L_{i,t} L_{i,t+1}^d
\]

where \( \Delta^L_{i,t} = \mathcal{G} \left( \frac{R_{i,t}^L(b)}{R_{i,t}^L} \right)^{-\epsilon_L} \) is the interest rate dispersion term and \( L_{i,t+1}^d \) is the aggregate demand from home and foreign entrepreneurs, and is defined by,

\[
L_{i,t+1}^d = \mathcal{G} \left( L_{h,i,t+1} (e) \right) + \mathcal{G} \left( L_{f,i,t+1} (e) \right)
\]

\[53\]
Recalling that entrepreneurs borrow to domestic and foreign banks with varieties \( b \) produced by wholesale branches, the equilibrium finally writes,

\[
\begin{align*}
L_{c,t+1}^s &= \left( 1 - \alpha_c^L \right) \left[ \frac{R_{c,t}^L}{P_{c,t}} \right]^{-\nu} L_{c,t+1} + \frac{n-1}{n} \alpha_p^L \left[ \frac{R_{p,t}^L}{P_{p,t}} \right]^{-\nu} L_{p,t} \Delta_L^L_{c,t} \\
L_{p,t+1}^s &= \left( 1 - \alpha_p^L \right) \left[ \frac{R_{p,t}^L}{P_{p,t}} \right]^{-\nu} L_{p,t+1} + \frac{n-1}{n} \alpha_c^L \left[ \frac{R_{c,t}^L}{P_{c,t}} \right]^{-\nu} L_{c,t} \Delta_L^L_{p,t}
\end{align*}
\]  

(A.54)

Aggregate loan rate index evolves according to,

\[
(R_{t,i}^L)^{1-\epsilon_L} = \theta_t^L (R_{t,i-1}^L)^{1-\epsilon_L} + (1 - \theta_t^L) (R_{t,i}^L)^{1-\epsilon_L}
\]  

(A.55)

A.9.3 Interbank Market

On the perfectly competitive interbank market, the market clears when the following condition holds,

\[
\begin{align*}
IB_{c,t+1}^s &= \frac{\lambda}{1 - \lambda} \left( 1 - \alpha_c^{IB} \right) \left[ \frac{R_{c,t}^{IB}}{P_{c,t}} \right]^{-\xi} IB_{c,t+1}^d + \frac{n-1}{n} \alpha_p^{IB} \left[ \frac{R_{p,t}^{IB}}{P_{p,t}} \right]^{-\xi} IB_{p,t}^d \\
IB_{p,t+1}^s &= \frac{\lambda}{1 - \lambda} \left( 1 - \alpha_p^{IB} \right) \left[ \frac{R_{p,t}^{IB}}{P_{p,t}} \right]^{-\xi} IB_{p,t+1}^d + \frac{n-1}{n} \alpha_c^{IB} \left[ \frac{R_{c,t}^{IB}}{P_{c,t}} \right]^{-\xi} IB_{c,t}^d
\end{align*}
\]  

(A.56)

A.9.4 Deposit Market

The equilibrium on deposit market is defined by the aggregate demand for deposits services of households and the aggregate supply from deposit packers. Aggregating the demand function from deposit packers leads to the equilibrium on this market,

\[
G (D_{i,t+1} (b)) = \Delta^D_{i,t} G (D_{i,t+1}^d (j))
\]  

(A.57)

where \( \Delta^D_{i,t} = G \left( \frac{R_{i,t}^{D0} (0)}{R_{i,t}^{D0}} \right)^{-\epsilon_D} \) is the interest rate dispersion term, while the aggregate deposit rate index evolves according to,

\[
(R_{i,t}^D)^{1-\epsilon_D} = \theta_t^D (R_{i,t-1}^D)^{1-\epsilon_D} + (1 - \theta_t^D) (R_{i,t}^D)^{1-\epsilon_D}
\]  

(A.58)

B Definitions for Higher-Order Approximations

B.1 The welfare index

In each country, we compute the fraction of consumption stream from alternative monetary policy regime to be added (or subtracted) to achieve the benchmark reference. The welfare of aggregate households in country \( i \) writes,

\[
W_{i,t} = \sum_{\tau=0}^{\infty} \beta^\tau U_i (C_{i,t+\tau}, H_{i,t+\tau}), \quad i = c, p
\]  

(B.1)
where the utility function is defined by,

\[ U(C_{i,t}, H_{i,t}) = e^{\epsilon_i \psi_i} \left( \frac{(C_{i,t} - h^c_i C_{i,t-1})^{1-\sigma^c}}{1 - \sigma^c} - \chi_i \frac{H^1_{i,t} + L^1_{i,t}}{1 + \sigma^L} \right) - \lambda^R (R_{t} - \bar{R})^2 \] (B.2)

Following Pariès et al. and Woodford, we account for the zero lower bound by adding to the utility function a term \( \lambda^R (R_{t} - \bar{R})^2 \) that makes the probability of hitting the zero lower bound shrink. Assuming that the Eurosystem authorities are concerned by the mean welfare of the two countries, we defined the welfare objective \( W_{u,t} \) of the monetary union by the arithmetical average according to the size of each area composing the union,

\[ W_{u,t} = nW_{c,t} + (1 - n) W_{p,t} \] (B.3)

We consider \( W_{b,t}^a \) and \( W_{a,t}^b \) the welfare indexes generated by policy regimes \( b \) and \( a \). Parameter \( \psi \) denotes the cost of leaving regime \( a \) for the regime \( b \) in terms of unconditional consumption for households populating the economy. The no-arbitrage condition between implementing the regimes \( a \) and \( b \) is determined by the following equality,

\[ W_{a,t}^b \left( C_{a,t+\tau}, H_{a,t+\tau} \right) = W_{a,t}^b \left( (1 - \psi) C_{a,t+\tau}, H_{a,t+\tau} \right), \quad \tau \geq 0 \] (B.4)

Where \( \psi \times 100 \) measures the welfare cost in percentage points of permanent consumption of leaving regime \( a \) for the regime \( b \).

Developing expression (B.4) leads to,

\[ W_{u,t}^a = (1 - \psi)^{1-\sigma^c} W_{a,t}^b + \left( (1 - \psi)^{1-\sigma^c} - 1 \right) \mathcal{H}_{u,t}^b \]

Where \( \mathcal{H}_{u,t}^b \) writes,

\[ \mathcal{H}_{u,t}^b = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \left\{ e^{\epsilon_i \psi_i} \chi_i \left( \frac{H_{a,t+\tau}}{1 + \sigma^L} \right)^{1+\sigma^L} + \lambda^R (R_{t+\tau} - \bar{R})^2 \right\} \]

Put in a recursive fashion, for each country, \( \mathcal{H}_{i,t}^b \) writes,

\[ \mathcal{H}_{i,t}^b = \chi_i e^{\epsilon_i \psi_i} \frac{H_{a,t}}{1 + \sigma^L} + \lambda^R (R_{t} - \bar{R})^2 + \beta \mathbb{E}_t \mathcal{H}_{a,t+1} \] (B.5)

To sum up ours calculations, we add in the model 4 equations. First, the equations that define the welfare index of home and foreign country are,

\[
\begin{align*}
W_{c,t} &= U_c (C_{c,t}, H_{c,t}) + \beta \mathbb{E}_t W_{c,t+1} \\
W_{p,t} &= U_p (C_{p,t}, H_{p,t}) + \beta \mathbb{E}_t W_{p,t+1}
\end{align*}
\] (B.6)
and equations that are required to convert welfare in terms of unconditional consumption are,

\[
\begin{align*}
H_{c,t} &= \chi_c e^{\beta R} \Gamma_{c^{1+\sigma}} + \lambda R (R_t - \bar{R})^2 + \beta \mathbb{E}_t H_{c,t+1} \\
H_{p,t} &= \chi_p e^{\beta R} \Gamma_{p^{1+\sigma}} + \lambda R (R_t - \bar{R})^2 + \beta \mathbb{E}_t H_{p,t+1}
\end{align*}
\] (B.7)

After solving the model under regime \(a\) and \(b\) using 10,000 draws generated by the model for each regime, we obtain the asymptotic mean \(E[\cdot]\) of \(H_{h,t}^a, H_{f,t}^a, W_{h,t}^a, W_{f,t}^a\). We measure the welfare cost by finding the value of \(\psi\) that solves,

\[
nE[W_{c,t}^a] + (1-n) E[W_{p,t}^a] = n \left( (1-\psi)^{1-\sigma^c} \left( E[W_{h,t}^a] + E[H_{h,t}^a] \right) - E[H_{h,t}^b] \right) + (1-n) \left( (1-\psi)^{1-\sigma^c} \left( E[W_{f,t}^a] + E[H_{f,t}^a] \right) - E[H_{f,t}^b] \right)
\] (B.8)

We cannot find an analytical solution of the problem, we use matlab solver to get the numerical solution.

On the other hand, the consumption loss \(\psi\) experienced by households living in country \(i\) is determined by,

\[
E[W_{i,t}^a] = (1-\psi)^{1-\sigma^c} \left( E[W_{i,t}^b] + E[H_{i,t}^b] \right) - E[H_{i,t}^b]
\] (B.9)

### B.2 New Keynesian Phillips curves in a recursive fashion

#### B.2.1 Sticky Price

Denoting by \(p_{i,t}^s\) the relative price defined by, \(p_{i,t}^s = P_{i,t}/P_{i,t}\), then we divide by \(P_{i,t}^{1-\epsilon_P}\), the aggregate price,

\[
1 = (1-\theta^P_i) \left( p_{i,t}^s \right)^{1-\epsilon_P} + \theta^P_i \left( \frac{\pi^P_{i,t-1}}{\pi^P_{i,t}} \right)^{1-\epsilon_P}
\] (B.10)

the dispersion term, \(\Delta^P_{i,t} = \mathcal{G} \left( \frac{P_{i,t}^s}{P_{i,t}} \right)^{-\epsilon_P}\), written in a recursive fashion is determined by,

\[
\Delta^P_{i,t} = (1-\theta^P_i) \left( p_{i,t}^s \right)^{-\epsilon_P} + \theta^P_i \left( \frac{\pi^P_{i,t}}{\pi^P_{i,t-1}} \right)^{\epsilon_P} \Delta^P_{i,t-1}
\] (B.11)

Denoting the real marginal cost \(mc_{i,t} = MC_{i,t}/P_{i,t}\) and \(p_{i,t}^s = P_{i,t}^s/P_{i,t}\), the first order condition of intermediate firms under \textit{ex ante} symmetry hypothesis becomes,

\[
\mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau \frac{\lambda^{C_{i,t+\tau}}}{\lambda_{i,t}^{C}} \left[ \left( \prod_{j=1}^{\tau} \frac{\pi^P_{i,t-1+j}}{\pi^P_{i,t+j}} \right)^{-\epsilon_P} p_{i,t}^s - \frac{\epsilon_P}{\epsilon_P-1} \left( \prod_{j=1}^{\tau} \frac{\pi^P_{i,t-1+j}}{\pi^P_{i,t+j}} \right)^{-\epsilon_P} mc_{i,t+\tau} \right] Y_{i,t+\tau} = 0
\]

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to simplify the equation, we split the sum,

\[
\begin{align*}
S^1_{i,t} &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta^P_i)^\tau \frac{\lambda_{i,t+\tau}}{\lambda_{i,t}} \left( \prod_{j=1}^\tau \frac{\pi_{i,t+j-1}}{\pi_{i,t+j}} \right)^{1-\epsilon_P} p^*_i Y_{i,t+\tau} \\
S^2_{i,t} &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta^P_i)^\tau \frac{\lambda_{i,t+\tau}}{\lambda_{i,t}} \left( \prod_{j=1}^\tau \frac{\pi_{i,t+j-1}}{\pi_{i,t+j}} \right)^{-\epsilon_P} \frac{\epsilon_P}{(\epsilon_P-1)} e^\epsilon_{i,t+\tau} mc_{i,t+\tau} Y_{i,t+\tau}
\end{align*}
\]

Thus we can write the sums \(S^1_{i,t}\) and \(S^2_{i,t}\) in a recursive fashion. Finally, the new Keynesian Phillips curve writes,

\[
\begin{align*}
S^1_{i,t} &= (p^*_i)^{1-\epsilon_P} Y_{i,t} + \beta \theta^P_i \mathbb{E}_t \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \left( \frac{\pi_{i,t+1}}{\pi_{i,t+1}} \right)^{1-\epsilon_P} \frac{p^*_{i,t+1}}{p^*_{i,t}} S^1_{i,t+1} \\
S^2_{i,t} &= (p^*_i)^{-1-\epsilon_P} Y_{i,t}mc_{i,t} \frac{p^*}{(\epsilon_P-1)} e^\epsilon_{i,t} + \beta \theta^P_i \mathbb{E}_t \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \left( \frac{\pi_{i,t+1}}{\pi_{i,t+1}} \right)^{-\epsilon_P} \frac{p^*_{i,t+1}}{p^*_{i,t}} S^2_{i,t+1} \\
S^1_{i,t} &= S^2_{i,t} \quad (B.12)
\end{align*}
\]

We use five equations ((B.10), (B.11) and (B.12)) to perform higher-order approximations of the sticky price model.

### B.2.2 Sticky Credit Rate

We divide by \((R^L_{i,t})^{1-\epsilon_L}\) the aggregate rate, we get,

\[
1 = \theta^D_i \left( \frac{R^L_{i,t}}{R^L_{i,t-1}} \right)^{1-\epsilon_L} + (1 - \theta^L_i) \left( \frac{R^L_{i,t}}{R^L_{i,t}} \right)^{1-\epsilon_L} \quad (B.13)
\]

the dispersion term, \(\Delta^L_{i,t} = \mathcal{G} \left( R^L_{i,t} (b) / R^L_{i,t} \right)^{-\epsilon_L} \), is defined by,

\[
\Delta^L_{i,t} = (1 - \theta^L_i) \left( \frac{R^L_{i,t}}{R^L_{i,t-1}} \right)^{-\epsilon_L} + \theta^L_i \left( \frac{R^L_{i,t}}{R^L_{i,t-1}} \right)^{-\epsilon_L} \quad \Delta^L_{i,t-1} \quad (B.14)
\]

Under \textit{ex ante} symmetry hypothesis, the first order condition of the bank allowed to reset its credit rates reads as follows,

\[
\sum_{\tau=0}^{\infty} (\theta^L_i)^\tau \frac{\lambda^L_{i,t+\tau}}{\lambda^L_{i,t}} \left[ R^L_{i,t} - \frac{\epsilon_L}{\epsilon_L - 1} e^\epsilon_{i,t+\tau} MC^L_{i,t+\tau} \right] \left( \frac{R^L_{i,t}}{R^L_{i,t+\tau}} \right)^{-\epsilon_L} L^d_{i,t+1+\tau} = 0
\]
to simplify the equation, we split the sum,

\[
\begin{align*}
B_{t,t}^1 &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_i^D) \frac{\chi_{i,t+\tau}^c}{N_{i,t}} R_{i,t}^L \left( \frac{R_{t+\tau}^L}{R_{t+\tau}^L} \right)^{-\epsilon_L} L_{i,t+1+\tau}^d \\
B_{t,t}^2 &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\beta \theta_i^D) \frac{\chi_{i,t+\tau}^e}{N_{i,t}} e_i^{i,t+\tau} MC_{i,t+\tau}^L \left( \frac{R_{t+\tau}^L}{R_{t+\tau}^L} \right)^{-\epsilon_L} L_{i,t+1+\tau}^d
\end{align*}
\]

Thus we can write the sums \(B_{t,t}^1\) and \(B_{t,t}^2\) in a recursive fashion. Finally, the new Keynesian Phillips curve for credit rates writes,

\[
\begin{align*}
B_{t,t}^1 &= L_{i,t+1}^d \left( \frac{R_{t+1}^L}{R_{t+1}^L} \right)^{\epsilon_L} R_{i,t}^{L*} + \beta \theta_i^D \mathbb{E}_t \frac{\chi_{i,t+1}^c}{N_{i,t}} \left( \frac{R_{t+1}^L}{R_{t+1}^L} \right)^{1-\epsilon_L} B_{t,t}^1 \\
B_{t,t}^2 &= \frac{\epsilon}{\epsilon - 1} e_i^{i,t} MC_{i,t}^L L_{i,t+1}^d \left( \frac{R_{t+1}^L}{R_{t+1}^L} \right)^{\epsilon_L} + \beta \theta_i^D \mathbb{E}_t \frac{\chi_{i,t+1}^e}{N_{i,t}} \left( \frac{R_{t+1}^L}{R_{t+1}^L} \right)^{-\epsilon_L} B_{t,t}^2
\end{align*}
\]

(B.15)

\[B.2.3\ \text{Sticky Deposit Rate}
\]

We divide by \((R_i^{D})^{1-\epsilon_D}\), the aggregate rate writes,

\[
1 = \theta_i^D \left( \frac{R_{i,t}^{D*}}{R_{i,t}^{D}} \right)^{1-\epsilon_D} + (1 - \theta_i^D) \left( \frac{R_{i,t}^{D*}}{R_{i,t}^{D}} \right)^{1-\epsilon_D}
\]

(B.16)

the dispersion term, \(\Delta_i^{D} = \mathcal{G} \left( \frac{R_{i,t}^{D*}}{R_{i,t}^{D}} \right)^{-\epsilon_D}\) is defined by,

\[
\Delta_i^{D} = (1 - \theta_i^D) \left( \frac{R_{i,t}^{D*}}{R_{i,t}^{D}} \right)^{-\epsilon_D} + \theta_i^D \left( \frac{R_{i,t}^{D*}}{R_{i,t}^{D}} \right)^{-\epsilon_D} \Delta_i^{D, t-1}
\]

(B.17)

Under \textit{ex ante} symmetry hypothesis, the first order condition of the bank allowed to reset its deposit rates reads as follows,

\[
\sum_{\tau=0}^{\infty} (\theta_i^D) \frac{\chi_{i,t+\tau}^c}{N_{i,t}} \left[ R_{i,t}^{D*} - \frac{\epsilon_D}{\epsilon_D - 1} e_i^{i,t+\tau} MC_{i,t+\tau}^D \right] \left( \frac{R_{t+\tau}^{D*}}{R_{t+\tau}^{D}} \right)^{-\epsilon_D} L_{i,t+1+\tau}^d = 0
\]

to simplify the equation, we split the sum,

\[
\begin{align*}
\mathcal{D}_{t,t}^1 &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_i^D) \frac{\chi_{i,t+\tau}^c}{N_{i,t}} R_{i,t}^{D*} \left( \frac{R_{t+\tau}^{D*}}{R_{t+\tau}^{D}} \right)^{-\epsilon_D} D_{i,t+1+\tau}^d \\
\mathcal{D}_{t,t}^2 &= \mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_i^D) \frac{\chi_{i,t+\tau}^e}{N_{i,t}} e_i^{i,t+\tau} \left( \frac{R_{t+\tau}^{D*}}{R_{t+\tau}^{D}} \right)^{-\epsilon_D} D_{i,t+1+\tau}^d
\end{align*}
\]

(B.16)
Thus we can write the sums $D_{1,t}$ and $D_{2,t}$ in a recursive fashion. Finally, the new Keynesian Phillips curve for deposit rates writes,

\[
\begin{align*}
D_{1,t} &= D_{1,t+1}^d \left( \frac{R_{t+1}^D}{R_t^D} \right)^{\epsilon_D} R_t^D \beta_t^D E_t \frac{X_{t+1}^i}{X_t^i} \left( \frac{R_{t+1}^D}{R_t^D} \right)^{1-\epsilon_D} D_{1,t+1} \\
D_{2,t} &= e_{\varphi_D}^D \epsilon_D^{-1} MC_{t+1}^D D_{1,t+1} \left( \frac{R_{t+1}^D}{R_t^D} \right)^{\epsilon_D} + \beta_t^D E_t \frac{X_{t+1}^i}{X_t^i} \left( \frac{R_{t+1}^D}{R_t^D} \right)^{-\epsilon_D} D_{1,t+1} \\
D_{1,t} &= D_{2,t}
\end{align*}
\]

(B.18)

### C Robustness Check to the Zero Lower Bound

In the quantitative simulations, we performed monetary policy coefficients optimization exercises without considering the perspective of hitting the zero lower bound (ZLB). In DSGE models, the zero lower bound has initially been addressed by McCallum (2000) and Woodford (2002). In our simulations, we optimize the policy coefficient on inflation using a welfare criterion, given our grid search over $[1, 3]$ we found that the optimal parameter that penalizes inflation is $\varphi^* = 3$, which is higher than the estimated coefficient. A high value of $\varphi^*$ increases the volatility of the nominal interest rate and, in turn, increases the probability of violating the zero lower bound.

In this section, we check whether the model conclusions rely on realistic scenarios by computing the ZLB probability. To do so, we first consider the estimated model and generate artificial series for the Euro area given the estimated standard deviations of shocks. To get the probability of hitting the zero lower bound using the model asymptotic properties, we generate a larger sample of 1,000 draws and we compute the number of times the interest rate is below 0. To compute a confidence interval, we reiterate the operation 100 times. Finally, we do this exercise for various values of $\varphi^*$.

Figure 12 reports the results. We find that on policy coefficients lying in the interval $(1, 3]$ for $\varphi^*$ is relevant as the ZLB is clearly below 2%, which is fairly low. Considering a larger grid search interval could lead to fallacious results. For instance, Quint and Rabanal (2013) explore monetary policy coefficients in the interval $(1, 5]$ without computing the zero lower bound probability, this could lead to fallacious results in terms of welfare ranking of alternative implementation schemes. In the same vein, Darracq-Pariès et al. (2011) consider wider intervals for monetary policy coefficients than Quint and Rabanal (2013) and they probably face a ZLB probability higher than 20% in their quantitative simulations.
Figure 6: Prior and posterior distributions of parameters.
Figure 7: Dynamic correlations of the main variables: observable variables (blue) and 90% confidence band interval generated by the estimated model centered around the asymptotic mean.
Entrepreneurs

\[ Q_tK_t = N_t + L_t \]

Goods Producers

\[ Y_t = f(K_t, H_t) \]

Capital Suppliers

\[ Q_tK_t = f(I_t, K_{t-1}) \]

Banks

\[ L_t \]

Loans \( L_t \)

Figure 8: Implementing the financial accelerator in a real business cycles model

Ex ante Investment Project \( \omega \)

\[ E_tg(\omega)R_{i,t+1}^KQ_{i,t}K_{i,t+1} \]

Ex post profits to share

\[ \text{Entrepreneur: } \eta^e_{i,t}(\bar{\omega}_{i,t}R_t^LQ_{i,t-1}K_{i,t} - R^L_tL_{i,t}) \]
\[ \text{Bank: } \eta^b_{i,t}R^L_tL_{i,t} \]

Else (not gainful)

Entrepreneur and bank: 0

Figures 9: Profit sharing between the entrepreneur and the bank

\[ f(\omega) \]

\[ \int_{\omega_{\min}}^{\omega^c} f(x) dx = 5\% \]
\[ \int_{\omega^c}^{+\infty} f(x) dx = 95\% \]

Figure 10: Pareto distribution of heterogeneous entrepreneurs’ projects \( \omega \in [\omega_{\min}, +\infty[ \)
Figure 11: Implementing the interbank market in a New Keynesian Framework

Figure 12: Zero lower bound probability for various values of the monetary policy reaction parameter to inflation generated by the model.