Credit Imperfections, Labor Market Frictions and Unemployment: a DSGE approach

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March 8, 2015

Abstract

We construct and estimate a new-Keynesian DSGE model, integrating sticky prices in goods market and frictions in labor and credit markets. A search and matching process in the labor market and a costly state verification framework in the credit market are introduced. Capital spending, vacancy posting costs and wage bill need to be paid in advance of production and thus require external financing in a frictional credit market. According to our theoretical model, we find that the procyclicality of the risk premium impacts the vacancy posting decisions, the wage and unemployment levels in the economy. Credit market frictions may be the source of lower posting vacancies and higher unemployment level. Asymmetric information pushes up wholesale firms' marginal costs, as well as hiring costs by a financial mark-up charged by financial intermediaries. This financial mark-up is then transmitted by these firms on prices. Thus, it affects their hiring behavior, as well as wage and employment levels in the economy. An empirical evidence is then presented by estimating dynamic responses of labor and credit markets variables to identified monetary and credit shocks, using a structural Bayesian VAR method. Notably, after a shock on the risk premium, we observe a hump-shaped increase of unemployment and an increase of real wages. According to our theoretical model, it represents the higher marginal costs incurred by wholesale-firms due to the increase of the financial mark-up.

\textit{JEL classification}: E24, E32, E44, E52

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Keywords: new-Keynesian model, labor and credit markets frictions, sticky prices, intensive and extensive margins, monetary and credit shocks

1 Introduction

The model is a monetary new-Keynesian model with asymmetric information in the credit market à la Bernanke et al. (1999) and a search and matching process in the labor market à la Mortensen and Pissarides (1994). We use the costly state verification (CSV) framework because of its tractability and the facility that it offers to embed informational frictions in a general equilibrium analysis. Moreover, it allows to illustrate how credit market imperfections alter the transmission of monetary policy.

Our model enables to better understand cyclical fluctuations in key labor market variables (unemployment, vacancies, hours worked per employee and wages) and in credit market central variables (risk premium and default rate). Indeed, since capital spending, wage bill and vacancy posting costs are assumed to be paid partially by external funds, the interaction between frictions in credit and labor markets are key to better analyze the propagation and amplification of shocks on principal variables of both markets.

According to our theoretical model, we find that the procyclicality of the risk premium (the cost of external over internal funds) impacts the vacancy posting decisions, the wage bill and unemployment levels in the economy. In period of downturns, the risk premium increases and the net worth of firms decreases. It increases their dependence on external funds, making job posting more expensive. So, less vacancies are posted and a higher equilibrium unemployment is obtained. Thus, asymmetric information in the credit market pushes up marginal costs and prices, as well as hiring costs by a financial mark-up, depending on the levels of monitoring cost and idiosyncratic shock threshold. This financial mark-up is made to overcome the agency problem between financial intermediaries and wholesale-good firms. But it will be charged in return by these firms on prices and will affect their hiring behavior, as well as the wage and employment levels in the economy.

The following chart sheds light on the causal relationships that we study and highlight in our theoretical model and in its estimation.
The evolution of unemployment rate, Baa-Aaa spread and default rate between 1970-Q1 and 2007-Q4 for the United-States (US) is represented. The unemployment rate is the ratio of civilian unemployed persons to the civilian labor force. The default rate is the default rate for Moody's rated US speculative-grade corporate bonds. The Baa-Aaa spread is the Moody's seasoned Baa-Aaa corporate bond yield. A degree of correlation is observed among variables, especially for the unemployment and the Baa-Aaa spread. The correlation among these two variables is equal to 0.76. Thus, the higher the unemployment rate is, the higher the Baa-Aaa spread is and conversely. For the default rate, the correlation is less explicit, due to plausible structural forces between 1971 and 1980. However, some periods of correlation exist: 1970-Q1 until 1971-Q2 (0.81), 1979-Q1 until 1985-Q4 (0.6) and from 1990 (0.32). In our paper, we demonstrate that the unemployment rate is in part determine by the evolution of credit spreads and default rate.

To give more precise insights of our work, an empirical evidence is finally presented by estimating dynamic responses of labor and credit markets variables to identified monetary and credit shocks, using a structural Bayesian VAR method. We find that both shocks have a clear and significant impact on labor market variables, namely unemployment, real wages, hours worked per employee, job creation and destruction rates.
2 Related literature

Our work is at the intersection of different lines of research. First, a number of research papers introduce search and matching frictions on labor markets in real business cycle (RBC) models or in new-Keynesian (NK) models. Other articles highlight the role of financial frictions for macroeconomic dynamics, without taking into account search and matching frictions on labor markets. Finally, more recent studies embody simultaneously frictions in labor and credit markets in partial equilibrium models or in dynamic stochastic general equilibrium (DSGE) models, to study interactions and implications of these two types of frictions.

The assumption of Walrasian labor markets is considered as a weakness of standard RBC and NK models. Indeed, these models do not take into account variations in the number of unemployed, the extensive margin that never changes. They allow only to study variations in hours worked per employee, the intensive margin. This may seem annoying to the extent that unemployment is an important indicator of performances of the economy in its use of resources and it is a major policy issue. Furthermore, this kind of models is ineffective to explain many stylized facts, such as the inertia of inflation together with the large and persistent response of output after a monetary policy shock or the propagation mechanism of output after a technological shock. They are not able also to study the effect of aggregate shocks on unemployment dynamics. As a consequence, many articles have introduced search and matching frictions in labor markets, based on Mortensen and Pissarides (1994) framework, in RBC models or in NK models (Merz (1995), Andolfatto (1996), Walsh (2005), Krause et al. (2008), Gertler and Trigari (2009), Thomas and Zanetti (2009), Trigari (2009), Christooffel et al. (2009), Lechthaler et al. (2010), Blanchard and Galí (2010), Galí et al. (2011), Campolmi and Faia (2011), Christiano et al. (2013)).

Papers, as those of Merz (1995) and Andolfatto (1996), study implications of search and matching frictions for economic fluctuations in a standard RBC model. Both model show that labor market frictions are a mechanism of amplification and persistence for technology shocks. These frictions improve the empirical performance of RBC models, compared to a standard one, even if they do not predict enough cyclical movements in vacancies and output compared to data. Moreover, Andolfatto (1996), by introducing extensive and intensive margins, finds that most of the variability of total hours worked is due to changes in unemployment level rather than hours worked per employee.

Then, several papers in the same spirit (Walsh (2005), Trigari (2009), Thomas and Zanetti (2009), Lechthaler et al. (2010) and Campolmi and Faia (2011)) examine the role of matching frictions in new-
Keynesian models. Walsh (2005) develops a new-Keynesian DSGE model with labor market frictions and with different potential sources of persistence (habit persistence, price stickiness and policy inertia) to see if it generates persistence in output and inflation after a monetary policy shock as observed in data. He found through calibration that it amplifies for US data the output response and decreases the inflation response to a monetary policy shock, as well as it generates persistence in output and inflation as observed in data and as standard NK models do not succeed to generate. Trigari (2009) considers cyclical fluctuations of output, inflation and labor market variables following a monetary shock. She studies the possibility of endogenous separation between firms and workers, as well as extensive and intensive margins. Her estimated model is able to replicate well for US data the observed responses of output, inflation and labor market data to a monetary policy shock. Using a VAR, she finds as observed in data that in a model with labor market frictions, the response of inflation is less volatile and response of output more persistent after a monetary policy shock than in a standard NK model. Finally, Campolmi and Faia (2011) study also the role of search and matching frictions in new-Keynesian model, but by adopting the framework of a small open economy. They find that real wages, marginal costs and profits are more sensitive to productivity and monetary policy shocks for countries with lower replacement rates.

However, the Mortensen-Pissarides search and matching model of unemployment remains unable to match important stylized facts observed in data. In particular, these types of models are not performing well to explain high volatility and persistence of unemployment, vacancies and market tightness (Shimer (2004) and Hall (2005)) and the relative smooth behavior of real wages found in data. The framework of wage Nash bargaining leads to an exaggerated procyclical movements in wages after a positive productivity shock for example, that dampens the firm’s incentives to hire. Wages absorb much of the change in the expected benefit to a new worker induced by fluctuations in labor productivity. As a consequence, several papers try to tackle this issue by introducing wage rigidity mechanisms (Shimer (2004), Gertler and Trigari (2009) and Christiano et al. (2013)), hiring and firing costs. First, Blanchard and Galí (2010) find that search and matching frictions modify the level of unemployment but the unemployment rate is invariant to productivity shocks. Thus, they study alternative wage-setting (Nash bargaining wage and more rigid real wages) and show that rigid wages enable to have inefficient fluctuations in unemployment after a productivity shock. Lechthaler et al. (2010) introduce in a new-Keynesian model labor market frictions, through hiring and firing costs but no wage rigidity. They find through a calibration on a given European country, more persistence in output and unemployment in response to real and monetary policy shocks.
shocks and in inflation in response to real shocks, as well as a strong amplification effect of these shocks on unemployment and on the job finding rate. Gertler and Trigari (2009) reproduce by calibration, in a standard Mortensen-Pissarides search and matching framework with a staggered multiperiod Nash wage, the relatively volatile behavior of unemployment and the relative smooth behavior of real wages over the business cycle as observed in data.

On the other hand, frictions have been also studied on the credit market side (Bernanke and Gertler (1989), Bernanke and Gertler (1995), Carlstrom and Fuerst (1998), Kiyotaki and Moore (1997), Bernanke et al. (1999), Carlstrom and Fuerst (2001), Paustian (2004) and Fiore and Tristani (2013)). They have been devoted to understand the relationship between financial markets and overall macroeconomic performances. Financial factors are indeed suspected to amplify and increase persistence of macroeconomic variables responses to aggregate shocks. The idea behind is that deteriorating credit conditions could be the source of poor economic activity and not the consequence of a declining real economy.

Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Bernanke et al. (1999) develop the concept of a financial accelerator in DSGE models integrating money and price stickiness. Without credit frictions, an entrepreneur can resort to external financing to raise capital at a risk-free interest rate. With credit market frictions, information asymmetry appears in the form of moral hazard between the lender and the borrower. Borrower will indeed be induced to report to the lender a lower real output produced than their true level. As a consequence, this type of asymmetric information can lead first to borrowing restrictions for borrowers on the amount of external financing available, based on the existence of collateral constraints to cover their potential inability to reimburse loans as in Kiyotaki and Moore (1997). Second, it can lead to a second type of financial frictions, namely a higher cost of external financing compared to internal financing opportunity cost (the risk-free interest rate), that to say an external finance premium or a risk premium, pays by entrepreneurs.

In a very similar spirit and few years before, ? introduce in a canonical RBC model the same kind of informational asymmetry between lenders and borrowers and show that it leads the economy to return more slowly to the steady-state after being hit by a shock (propagation mechanism) and leads to less amplification because agency costs create an endogenous mark-up in an output model. The mark-up distorts factor markets, so wages and capital rental rates are below their corresponding productivities. Thus, increases in net worth lower agency costs and hence the mark-up. Debt arises as the optimal financial contract between firms and banks and firms must borrow at a premium over the risk-free rate. The finan-
cial contract is designed to minimize the expected agency costs. It specifies the returns when bankruptcy or success occurs and a monitoring threshold (for reported profits below the threshold, the lender pays the state verification costs and above the threshold, the lender does not pay to audit the project result). The threshold is a decreasing function of borrower’s capital and an increasing function of the deposit risk free-rate (opportunity cost).

Fiore and Tristani (2013) show, by adopting the costly state verification set-up, that financial market conditions are important to explain macroeconomic outcomes because it affects firms’ marginal costs. Higher credit spreads increase lending rates and marginal cost of credit for firms, which lead to increase prices and as a consequence, it affects output.

All these papers assume standard Walrasien labor markets. Only few papers consider both credit and labor markets frictions, as the ones of Thomas and Zanetti (2009), Christiano et al. (2011), Petrosky-Nadeau (2014) and Zanetti and Muntaz (2013). Labor market frictions imply that it is costly to hire new workers. The functioning of labor markets prevent the competitive allocation of labor resources, and thus it will interact with financial frictions to impact production, unemployment, investment and capital accumulation. Those models enhance the Bernanke et al. (1999) framework with a more realistic labor market. Christiano et al. (2011) show in a new-Keynesian model that financial and employment frictions are able to change the model dynamics in an open economy setting, and improve the forecasting properties of the model for Swedish data, in particular for inflation. Thomas and Zanetti (2009) makes out that financial shocks are important to explain business cycles fluctuations, because they impact firm’s ability to raise funds and amplify/dampen the response of macroeconomic variables (such as unemployment, wages and vacancy posting) to shocks. Petrosky-Nadeau (2014) considers that firms finance only their vacancy costs with external financing on frictional credit markets. He finds that the easing of financing constraints during an expansion (a productivity shock) reduces the opportunity cost for resources allocated to job creation (cost channel), because firms are able to accumulate net worth. Credit market frictions generate persistence in the dynamics of labor-market tightness and have a moderate effect on amplification. Zanetti and Muntaz (2013) demonstrate that labor and financial frictions are supported by the data and that they play together to amplify or reduce the variables’ reaction to shocks.

Note that our framework is different from the one of Acemoglu (2001) or Wasmer and Weil (2004), Petrovsky-Nadeau and Wasmer (2014), who assume search frictions on both labor and credit markets.
Search frictions and agency costs are both credit market imperfections but we decide to focus on agency costs by a costly-state verification framework. Wasmer and Weil (2004) find that labor and credit market frictions work together to amplify macroeconomic volatility. Note also that our work is included in the spirit of researches about the impact of credit market imperfections on investment flows but we decide to focus on their impact on employment flows, hours worked and wages.

3 The model

3.1 Model overview and timing summary

The model is populated by seven types of agents: households, wholesale-good firms managed by entrepreneurs, retailers, final-good firms, a representative bank and a government that conducts monetary and fiscal policies.

The household sector is represented by a continuum of identical households of length unity. Each household is constituted of members who are either working or unemployed. All members are supposed to be risk-averse. They supply labor, consume, rent capital and save through money holding and through their deposits in a financial intermediary.

Entrepreneurs are risk-neutral and have finite lifetime. Following Bernanke et al. (1999), each entrepreneur is assumed to have a given probability to survive to the next period. They manage wholesale firms, that produce wholesale goods using a constant return-to-scale technology using labor and capital as inputs. Surviving entrepreneurs carry their profits as a part of their net worth. Dying entrepreneurs consume everything. After deciding on the number of new workers they need, entrepreneurs, based on their net worth amount and their expected production and returns, borrow funds from banks to post vacancies (and recruit workers), to pay capital spending and the wage bill in advance. However, wholesale-good production is subject to an idiosyncratic shock, privately observed by entrepreneurs, while banks need to pay a monitoring cost to check the real output produced, as well as the efficiency of the recruitment process. This agency problem will alter the real recruitment cost and the marginal cost of production for wholesale firms.

As soon as funds are obtained, entrepreneurs enter the labor market, match with their potential employees and bargain on wages. Whenever it happens, the match is assumed to keep going on until it is exogenously costlessly destroyed.

Finally, the production sector has two different layers as in Bernanke et al. (1999). At the first layer, where
agency problem and search and matching frictions occur, a continuum of perfectly competitive wholesale firms produce a homogeneous good using capital and labor as explained before. At the second layer, where price stickiness arises, wholesale goods are differentiated costlessly by a continuum of monopolistic firms. The final good is then homogeneous and can be used for consumption, investment and government spending. Profits from retailers are rebated lump-sum to households.

The seven agents are thus interacting in six different markets (labor market, capital market, credit market, liquidity market, wholesale-good market, intermediate-good market and final-good market), where the timing of events is given by:

0- A given fraction of entrepreneurs born so as to ensure a constant fraction of entrepreneurs at each period in the economy. An exogenous amount is given to entrepreneurs, to be sure that they all continue to be entrepreneurs.

1- Monetary policy and aggregate shocks are realized. Liquidity market opens.

2- Given all expected prices and revenues in the economy, households decide on their level of consumption, deposits, money holding, investment and on the capital rate of utilization.

3- Credit market opens: banks accumulate the deposited amounts by households at the end of period \( t - 1 \) in order to grant them as loans at the beginning of the current period \( t \). The credit market clears when the amount of deposits equals the amount of granted loans. Entrepreneurs own and manage the wholesale production sector. They enter period \( t \) with a net worth, either composed by the exogenous endowment and the accumulated net worth at the end of period \( t - 1 \) for the last period solvent entrepreneurs. Or they enter the period \( t \) with a net worth given by the exogenous endowment for the entrepreneurs who went bankrupt last period. They all borrow from banks using a nominal financial contract, in order to cover their expected production bill (labor and capital costs) and vacancies posting costs. Indeed, they are assumed to be paid in advance.

4- Labor market opens: entrepreneurs post vacancies at a real unit cost and recruit a given number of workers. Then, the wage is established after a Nash bargaining process and new hired employees start
working immediately. The Nash bargained wage and the vacancy posting costs have to be paid immediately by entrepreneurs, using their loans.

5- Capital market opens: households own capital. They rent effective capital to entrepreneurs at a perfectly competitive price. They accumulate capital at the end of the period $t-1$ to lend it during the current period $t$ to entrepreneurs. The capital market clears when the amount of effective capital supplied by households is equal to the amount demanded by entrepreneurs. Entrepreneurs have to pay immediately their capital costs to households, using their loans.

6- Wholesale-good market: wholesale goods are produced by wholesale-good firms thanks to labor and capital. After wholesale production occurs, entrepreneurs sell it to retailers and declare either being solvent or bankrupt, after having observed privately their own idiosyncratic shocks. Solvent entrepreneurs, characterized by a sufficiently high idiosyncratic shock, pay back their loan and keep the remaining amount to use it at the end of the period $t$ (to consume and/or to accumulate net worth). The bank spends a monitoring cost, proportional to the realized firm’s value, in order to check the output produced by bankrupt entrepreneurs and confiscates the proceeds of production left by these entrepreneurs. These later can neither consume nor carry over net worth to the coming period. Then, banks reimburse households deposits. The role of the financial intermediary is well defined: it allows to mitigate the monitoring cost and to avoid its duplication.

7- Intermediate-good market: retailers are a set of monopolistically competitive firms owned by households. They buy the wholesale good and differentiate it costlessly. However, only a given fraction of retailers are able to fully re-optimize their prices. All the realized profits are transferred to households, the owners of retailers, at the end of the period $t$.

8- Final-good market: they re-sell final goods to households (to consume and to invest), to the government (government spending) and to solvent exiting entrepreneurs, at a consumer’s price index.

9- Households consume, invest, make their deposits and their money holding.

10- Solvent entrepreneurs decide either on their consumption or on their net worth, depending on their
probability of death in the current period $t$: those exiting the economy at the end of the period $t$ consume all their net worth just before death, and those keeping in the economy will accumulate totally their net worth thanks to money.

11- All markets clear.

Figure 3 reports the flow of funds between agents.
Figure 2: Model overview and flows of funds
3.2 Households

Households consume, save through deposits, hold money, invest in capital and supply passively labor to entrepreneurs. They transfer wealth form a period to another by holding money. At the beginning of the period $t$, given all expected prices and revenues in the economy, households decide how much they want to consume, to save through deposits and money and how much they want to invest final goods to accumulate capital. This capital will be rented at the next period $t+1$ to entrepreneurs on a perfectly competitive capital market. Households decide also the capital utilization rate for the current period $t$, which will determine the effective capital submitted for rent in this current period $t$. To decide, households expect as resources, wages earned after the matching process of the period $t$ by workers and unemployment benefits earned by unemployed at the end of period $t$. They earn also incomes from the rental of capital and profits from retailers, as they own them. Finally, they will receive from banks, after reimbursements of wholesale-good firms loans, the last period risk-free interest rate, multiplied by the amount of deposits decided last period. This revenue is sure because financial intermediation is assumed to be realized through a large number of atomistic risk-neutral banks, holding enough large and diversified portfolios to ensure perfect risk pooling for their main creditors, the households.

Then, when the labor market opens, unemployed households members supply labor to entrepreneurs passively. A fraction of them is matched with entrepreneurs and begins to work immediately. The other fraction is not matched and stays unemployed. Newly matched workers and workers who have been matched without destruction in the period $t-1$ receive their wages immediately after the wage bargaining. The unemployed have to wait the end of the period to receive unemployment benefits. At the end of the period, after the production of final goods, households have to pay lump-sum taxes to the government, that will finance unemployment benefits. Finally, households consume effectively, invest final goods to accumulate capital, hold money and make their deposits to banks.

Employed and unemployed members

After obtaining a loan from the representative bank, a wholesale firm $i$ posts actively $V_{it}$ job vacancies at a real unit cost, $\gamma$, to attract unemployed workers, $U_t$, who are searching passively for a job.

**Assumption 1.** Only the unemployed workers can search passively for a job and can be hired. Current employed workers are not allowed to look for another job. Job-to-job transition is not considered.
Job creation occurs when an entrepreneur and an unemployed worker meet on the labor market after a search and matching process à la Mortensen and Pissarides (1994) and agree on a Nash bargaining wage. At the end of this process, a wholesale firm \( i \) employs \( n_{it} \) workers at a real hourly wage \( W_t \). As there exists a continuum of wholesale firms represented by the unit interval, the total number of vacancies and the total number of employed workers are:

\[
V_t = \int_0^1 V_{it} \, di \\
n_t = \int_0^1 n_{it} \, di
\]

**Assumption 2.** There is a full participation of workers. They are either employed or unemployed workers looking for a job. The transition between in and out the labor force is ignored.

Formally, total vacancies, \( V_t \), will be filled by unemployed workers, \( U_t \), via an aggregate constant return to scale matching function, \( M(U_t, V_t) \), specified by\(^1\):

\[
M(U_t, V_t) = \epsilon_t^M U_t^\rho V_t^{1-\rho}
\]

where \( \rho \in (0, 1) \) is the elasticity of matches to unemployment and \( \epsilon_t^M \) is the matching process efficiency shock. It follows a first-order auto-regressive process:

\[
log(\epsilon_t^M) = (1 - \rho_M)log(\epsilon^M) + \rho_Mlog(\epsilon_{t-1}^M) + u_t^M, \quad \rho_M \in [0, 1) \text{ where } u_t^M \overset{iid}{\sim} N(0, \sigma_M^2)
\]

\( \epsilon^M \) is the steady-state value of the matching process efficiency. Moreover, as standard in the literature, the matching technology is assumed to be concave and increasing in both arguments.

**Assumption 3.** As soon as the matching happens, new hired workers start working immediately\(^2\). Employed workers for whom the matching ends exogenously during period \( t \) (as explained below) are allowed

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\(^1\)Gertler et al. (2008) use the same specification. The Cobb-Douglas matching function is used in almost all macroeconomic models with search and matching frictions. Furthermore, the constant returns to scale assumption (homogeneity of degree one) seems to be supported empirically (Petrongolo and Pissarides (2001)).

\(^2\)Following Krause and Lubik (2007), Gertler et al. (2008), Thomas and Zanetti (2009) and Blanchard and Gali (2010), workers are assumed to be immediately productive after being hired.
to search for a new job in the same period. But a new matching is only possible after financial contracts and loans are decided and obtained, so only at the opening of the labor market at the next period.

The ratio of total vacancies to unemployed workers defines the labor market tightness from the firm point of view, \( \theta_t \), so that:

\[
\theta_t \equiv \frac{V_t}{U_t}
\]

The probability for a firm to fill a vacancy job during the matching process of the period \( t \), \( p_t \), is given by:

\[
p_t = \frac{M(U_t, V_t)}{V_t} = M(\theta_t^{-1}, 1)
\]

And the probability for an unemployed worker to find a job during the matching process of the period \( t \), \( q_t \), is given by:

\[
q_t = \frac{M(U_t, V_t)}{U_t} = M(1, \theta_t)
\]

Therefore, the number of hired workers during the matching of the period \( t \), \( \psi_t \), is given by:

\[
\psi_t = p_t V_t = M(U_t, V_t)
\]

Note that \( q_t = \theta_t p_t \) and \( \partial p_t / \partial \theta_t < 0 \), \( \partial q_t / \partial \theta_t > 0 \). The higher vacancy posts on unemployment (or the higher the labor market tightness from the firm point of view), the higher the probability for an unemployed worker to find a job and the lower the probability for a firm to fill a job. Both workers and firms take \( q_t \) and \( p_t \) as given.

**Assumption 4.** A job is assumed to be destroyed at an exogenous rate, \( \delta_t \), which evolves according to a first-order auto-regressive process:

\[
\log(\delta_t) = (1 - \rho_\delta) \log(\delta) + \rho_\delta \log(\delta_{t-1}) + u_t^\delta, \quad \rho_\delta \in [0, 1) \quad \text{where} \quad u_t^\delta \sim iid \ N(0, \sigma_\delta^2)
\]

\( \delta \) is the steady state value of \( \delta_t \). The destruction of a match can occur between the end of the matching period and the beginning of the next matching period. Many papers on searching and matching literature are considering the same assumption. See Blanchard and Galí (2010) for example.
Finally, the total number of employed workers after the matching process of the period \( t \), \( n_t \), is defined as the sum of the surviving workers from the exogenous separation at the end of period \( t-1 \), \((1-\delta_{t-1})n_{t-1}\), and the new hires from the matching of the period \( t \), \( \psi_t \):

\[
n_t = (1-\delta_{t-1})n_{t-1} + \psi_t
\]  

(5)

As the labor force is normalized to one, the number of unemployed after the matching process of the period \( t \) is:

\[
U_t = 1 - n_t
\]  

(6)

Since workers who discontinue their match during the period \( t \) are assumed to be allowed to search passively for a new job in the same period, the number of searching workers at the end of the period \( t \) is \( 1-(1-\delta_t)n_t \).

**Household behavior**

Households are seen as a large representative family, consisting of a continuum of members, represented by the unit interval and composed of workers who are either employed or unemployed. As in Merz (1995) and Andolfatto (1996), there is a full risk sharing of consumption and savings in order to avoid distributional issues due to heterogeneity in incomes. So, the family pools its income such that a perfect consumption is fully insured for all members. The same notation is used for the consumption of the representative household and for the consumption of each member\(^4\).

After a costly and time-consuming search and matching process à la Mortensen and Pissarides (1994) on the labor market, the number of employed household members is \( n_t \in (0,1) \), whereas the remaining members, \( 1-n_t \), are unemployed. Each employed worker supplies hours of labor at the real hourly wage \( W_t \). The real wage is determined through a decentralized Nash bargaining between matched couples of entrepreneurs and households. The representative household bargains with each entrepreneur separately, by taking as given wages in all other matches. Furthermore, entrepreneurs set unilaterally effective hours

\(^4\)The family optimally allocates the same consumption for each member, regardless their individual income.
of work, \( H_t \), at the time of the financial contract establishment. Therefore, the total number of hours worked by a representative household, \( N_t \), is given by:

\[
N_t = n_t H_t
\]

In addition to the real wage income, \( W_t N_t \), earned by employed workers and real unemployment benefits, \( \theta b \), received by unemployed workers, the representative family has a diversified ownership stake in monopolistic retailers, paying out a nominal lump-sum profit, \( \Pi \). He receives from banks the last period risk-free interest rate multiplied by the amount of deposits decided last period. Then, households hold an amount \( M_{t-1} \) of money carried from the previous period. The household makes also a nominal lump-sum transfer of taxes (or subsidies) for an amount of \( T_t \) to the government and rent effective capital, \( K_t \), to firms at a given real rate of \( r^K \). Finally, this total income is used to consume an amount \( C_t \) of final goods, to invest, \( I_t \), to save a nominal amount, \( D_t \), of bank deposits remunerated at a risk free rate, \( R_t \), and to accumulate an amount of money \( M_t \). Thus, the representative household budget constraint is given by:

\[
\frac{M_t - M_{t-1}}{P_t} + C_t + I_t + \frac{D_t}{P_t} \leq n_t W_t H_t + (1 - n_t)b + \frac{R_{t-1}D_{t-1}}{P_t} + \nu^K \nu_t K_{t-1}^p - \Upsilon(\nu_t)K_{t-1}^p + \frac{\Pi_t}{P_t} + \frac{T_t}{P_t}
\]

where \( P_t \) is the nominal aggregate price level.

Households own the economy’s stock of physical capital and choose the capital utilization rate, \( \nu_t \), which transforms physical capital, \( K^p_t \), into effective capital, \( K_t \). So the amount of effective capital that households can rent to wholesale firms is:

\[
K_t = \nu_t K_{t-1}^p
\]

\( \Upsilon(\nu_t) \) is the real cost of capital utilization per unit of physical capital, specified to match the following conditions at the steady state: \( \nu_t = 1 \), \( \Upsilon(1) = 0 \) and \( \frac{\Upsilon'(1)}{\Upsilon'(1)} = \eta_\nu \).

By investing \( I_t \) units of consumption goods during period \( t \), the representative household increases the nominal aggregate price level.

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\( b \) can be interpreted as home production or as unemployment benefits, as we do, provided by the government and financed by lump-sum taxes.
capital stock, $K^p_t$, available during period $t + 1$ according to:

$$K^p_t = (1 - \delta_K)K^p_{t-1} + \epsilon^I_t \left[ 1 - \Lambda \left( \frac{I_t}{I_{t-1}} \right) \right] I_t$$  \hspace{1cm} (9)

where $\delta_K$ is the capital depreciation rate and $\Lambda$ is the adjustment cost function, with $\Lambda(\gamma_\omega) = \Lambda'(\gamma_\omega) = 0$ and $\Lambda''(\gamma_\omega) = \nu_K > 0$, with $\gamma_\omega$ the economy’s steady state growth rate. $\epsilon^I_t$ is an investment-specific technological shock affecting the efficiency with which consumption goods are transformed into capital, following an auto-regressive process:

$$\log(\epsilon^I_t) = \rho^I \log(\epsilon^I_{t-1}) + u^I_t, \hspace{1cm} \rho^I \in [0, 1) \text{ where } u^I_t \sim N(0, \sigma^2_I)$$

Conditional on $\{H_t, n_t\}_{t=0}^\infty$ and taking as given the set of prices $\{P_t, W_t, R_t, r^K_t\}_{t=0}^\infty$, the household chooses the streams of optimal consumption of final goods $\{C_t\}_{t=0}^\infty$, of nominal money balances $\{M_t\}_{t=0}^\infty$, of deposits amount $\{D_t\}_{t=0}^\infty$, of investment $\{I_t\}_{t=0}^\infty$, of capital utilization rate $\{\nu_t\}_{t=0}^\infty$ and the stream of physical capital $\{K^p_t\}_{t=0}^\infty$, maximizing the following discounted utility function $^6$ subject to the budget constraint (7) and the capital accumulation constraint (9):

$$E_0 \sum_{t=0}^\infty \beta^t \left[ \epsilon^C_t \ln(C_t - hC_{t-1}) + \xi \ln \left( \frac{M_t}{P_t} \right) - \epsilon^H_t \frac{H^{1+\tau} n_t}{1 + \tau} \right]$$  \hspace{1cm} (10)

where $0 < \beta < 1$ is the intertemporal discount factor, $h$ is a habit persistence parameter, $\epsilon^C_t$ is a consumption preference shock, $\epsilon^H_t$ is a shock to supplied hours and $\tau$ denotes the inverse Frisch intertemporal elasticity of labor supply to the hourly real wage. When $h > 0$, the model allows for habit persistence in consumption preferences, in order to take into account the necessary empirical persistence in the consumption process.

Preference and hours supply shocks obey to the following stochastic processes:

$$\log(\epsilon^C_t) = \rho^C \log(\epsilon^C_{t-1}) + u^C_t, \hspace{1cm} \rho^C \in [0, 1) \text{ where } u^C_t \sim N(0, \sigma^2_C)$$

$$\log(\epsilon^H_t) = (1 - \rho^H) \log(\epsilon^H) + \rho^H \log(\epsilon^H_{t-1}) + u^H_t, \hspace{1cm} \rho^H \in [0, 1) \text{ where } u^H_t \sim N(0, \sigma^2_H)$$

where $\epsilon^H$ is the steady state value of $\epsilon^H_t$.

$^6$The form of the utility function is based on the one used by Gertler et al. (2008), Blanchard and Galí (2010) and Christiano et al. (2011).
The first-order conditions of the representative household’s problem are given by:

\[(C_t) \quad \lambda_t = \frac{\epsilon_t^C}{C_t - hC_{t-1}} - \beta h E_t \frac{\epsilon_{t+1}^C}{C_{t+1} - hC_t} \quad (11)\]

\[(D_t) \quad 1 = \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \right] \quad (12)\]

\[(v_t) \quad r^K_t = \Upsilon'(v_t) \quad (13)\]

\[(I_t) \quad \epsilon_t^I Q_t \left[ 1 - \Lambda \left( \frac{I_t}{I_{t-1}} \right) \right] = 1 + \epsilon_t^I Q_t \frac{I_t}{I_{t-1}} \Lambda' \left( \frac{I_t}{I_{t-1}} \right) - \beta E_t \epsilon_{t+1}^I \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} \left( \frac{I_{t+1}}{I_t} \right) \Lambda' \left( \frac{I_{t+1}}{I_t} \right) \quad (14)\]

\[(K^P_t) \quad Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[ (1 - \delta K) Q_{t+1} + r^K_{t+1} \nu_{t+1} - \Upsilon(\nu_{t+1}) \right] \quad (15)\]

\[(M_t) \quad \xi P_t = \frac{M_t}{\pi_{t+1}} \quad (16)\]

where \(\lambda_t\) is the Lagrange multiplier associated to the household’s budget constraint, \(Q_t\) is the marginal value of installed capital in consumption units and \(\pi_t \equiv \frac{P_t}{P_{t-1}}\) is the inflation rate.

### 3.3 Wholesale-good firms

Wholesale-good firms, indexed by \(i \in [0, 1]\), need labor and capital to produce. \(Y_{it}^{ws}\) is the quantity of wholesale goods produced by a firm \(i\) using \(N_{it}\) hours of labor and \(K_{it}\) units of effective capital, according to the following production function:

\[Y_{it}^{ws} = A_t K^\alpha_{it} N^{(1-\alpha)}_{it} \quad (17)\]

where \(A_t\) is the aggregate technology shock, realized at the beginning of each period, source of systematic risk. This shock is supposed to be stationary and evolves according to:

\[\log(A_t) = \rho_A \log(A_{t-1}) + u^A_t, \quad \rho_A \in [0, 1] \text{ where } u^A_t \overset{iid}{\sim} N(0, \sigma^2_A)\]

Effective capital, \(K_{it}\), is rented from households at a perfectly competitive price, \(r^K_t\). Total hours worked, \(N_{it}\), are paid to employed workers through wages, \(W_t N_{it}\). Needed new hires, \(\psi_{it}\), are obtained through a matching process on the labor market, implying vacancy posting costs, \(\gamma V_{it}\). The expected production bill, \(W_t N_{it} + r^K_t K_{it}\), and vacancy posting costs, \(\gamma V_{it}\), are assumed to be paid prior to production.

Each wholesale firm is managed by a finite lived risk-neutral entrepreneur, who may die at each period with a probability \((1 - \zeta)\). This assumption is made to be sure that entrepreneurs will not accumulate net
worth and will borrow from banks at each period\textsuperscript{7}. Intra-period entry into and exit out of wholesale firms at each period are ruled out. Entrepreneurs have a net worth, composed by a real exogenous endowment, $\chi_t$, and for the one solvent and not exiting the economy last period, by the net worth accumulated from the previous period. But this net worth will not be sufficient to cover the total production bill, as well as the vacancy posting costs. Entrepreneurs have to borrow from banks. Thus, a financial intermediation is realized through a large number of atomistic risk-neutral banks. Banks receive deposits from households at the end of period $t - 1$, that they use to lend to entrepreneurs in period $t$. Furthermore, entrepreneurs are subject to idiosyncratic shocks, privately observed by them, but not observed by banks. So banks have to monitor wholesale firms, which declare themselves bankrupt after the production occurs. The monitoring is costly and has to be integrated in the financial contract. An optimal financial contract will be determined between banks and entrepreneurs, maximizing the entrepreneurs expected returns, subject to the bank’s participation constraint.

3.3.1 Hiring decision

Before the financial contract establishment, the number of needed new hires, $\psi_{it}$, is determined by the human resources department of each firm. The wholesale firm consists of different departments. There is a human resources department, which is in charge with the recruitment process, and a management department (directed by the entrepreneur), which is inter alia in charge with the borrowing process. The human resources department decides on the labor needs of the firm, that determines the price of a new worker, or the potential value of this new worker for the firm. Each new worker is thus evaluated through this value, by assuming that the bank will be able to sell the worker to another firm in case of firm bankruptcy. It will enable then the management department to go to negotiate to the bank, based on this fictive worker price. At the end, the recruitment price of a new worker or the replacement cost of a worker will be obtained. So, just before the financial contract establishment, wholesale firms set up their hiring decision, based on the firms expected net returns. They determine the number of new workers, $\psi_{it}$, they have to take on, knowing the given probability for a firm to fill a vacancy job, $p_t$. Secondly, the hiring section of the firm post vacancies, $V_{it}$, on the labor market at the real unit cost $\gamma$, partially financed externally on a frictional credit market. It will thus determine the level of $Z_{it}$, the total cost of hiring a

\textsuperscript{7}The same assumption is made by Bernanke et al. (1999) and Paustian (2004). Carlstrom and Fuerst (1998) make the different assumption, that consumers and entrepreneurs have different time-discount factors, with entrepreneurs less impatient than consumers.
First, the hiring decision of a wholesale firm $i$ is the optimal solution to the following bellman equation:

$$J_{it} = \max_{\psi_{it}} \frac{P^{ws}_t}{P_t} Y^{ws}_{it} - W_t N_{it} - \frac{Z_t}{P_t} \psi_{it} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{it+1}$$

subject to $n_{it} = (1 - \delta_{t-1}) n_{i,t-1} + \psi_{it}$

$$N_{it} = n_{it} H_{it}$$

where $Z_t$ is the total cost or value of hiring a new worker, $P^{ws}_t$ is the wholesale-good price and $\frac{\beta \lambda_{t+1}}{\lambda_t}$ is the firm’s discount factor.

Normally, by taking as given the wage schedule, $W_t$, the hours of work per employee, $H_{it}$, the consumer price index, $P_t$, and the wholesale-good price, $P^{ws}_t$, a wholesale firm chooses first the number of hirings, $\psi_{it}$, and consequently the number of employees, $n_{it}$, so as to maximize its discounted value of future profits. However, an univariate optimization problem can be obtained in $n_{it}$ by embedding both constraints into the problem, since by choosing $n_{it}$, the firm determines implicitly $\psi_{it}$. So that the following first-order condition can be derived:

$$Z_t = \frac{P^{ws}_t}{P_t} \frac{(1 - \alpha) Y^{ws}_{it}}{n_{it}} - W_t H_{it} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} J_{n_{i,t+1}}$$

Using the envelop theorem, one obtains:

$$J_{n_{t-1},it} = (1 - \delta_{t-1}) \frac{Z_t}{P_t}$$

By taking equation (20) one period forward and plugging it in equation (19), the following Euler equation is derived:

$$Z_t = \frac{P^{ws}_t}{P_t} \frac{(1 - \alpha) Y^{ws}_{it}}{n_{it}} - W_t H_{it} + (1 - \delta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{Z_{t+1}}{P_{t+1}}$$

The expected cost of hiring a new worker is equal to the real expected value of a match. The real marginal value of a new hire is the sum of the net real return of a worker, which corresponds to its marginal productivity net of the marginal labor cost that is defined as the hourly wage, plus the real value of the

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\[8\] Since firms are owned by households, profits are evaluated in terms of utility brought to them.
continued relationship with the same worker, realized with the probability $(1 - \delta_t)$. Therefore, the hiring decision of firms is such that the expected profit from maintaining a match with an existing worker is equal to the expected costs of posting a new vacancy. These expected costs are now determined through the financial contract establishment.

### 3.3.2 Optimal financial contract

A financial contract is established between a financial intermediary and an entrepreneur, who needs to pay in advance its production bill and vacancy posting costs. The financial intermediation is realized through a large number of atomistic risk-neutral banks. Banks are assumed to hold enough large and diversified portfolios to ensure perfect risk pooling for their main creditors, the households, carrying deposits to banks.\(^9\)

Although we use the same costly state verification framework (CSV) used in BGG, Carlstrom and Fuerst (1998) and others, asymmetric information is introduced between wholesale-good producers, called entrepreneurs, and banks. Financial intermediaries and banks are used interchangeably in the model. They are operating in a competitive market, so that only the behavior of a representative bank will be considered below.

Furthermore, in order to eliminate aggregate uncertainty from the lender-borrower relationship, the aggregate technology shock, $A_t$, is assumed to be observed by all agents in the economy and it is realized before any loan contract is established. On the other hand, an idiosyncratic shock, $\omega_{it}$, is privately observed by the entrepreneur $i$ after the production takes place and it can be verified by the lender only at a monitoring cost, $\mu_t$, proportional to the realized value of the firm. Thus, this private information creates a moral hazard problem as the entrepreneur may be encouraged to under-report the true value of its production, when it has to reimburse the loan after the production occurrence.

Thus, under a costly state verification framework, the perfectly competitive financial intermediaries’ setting ensures that each firm-bank pair will write the borrowing contract that maximizes the expected return of the borrower, the lender, under the constraint that the expected return to the lender, the bank,

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\(^9\)Infinitely-lived households are risk averse, but they become risk neutral for the financial contract. Carlstrom and Fuerst (1998) explain this fact by the absence of uncertainty about the term of the one-period contract since the aggregate uncertainty is realized before the contract establishment. Furthermore, by the law of large numbers as banks are financing a continuum of different entrepreneurs, households know they will receive the expected return of the idiosyncratic shock.
exceeds its opportunity cost, namely the risk free interest rate, $R_t$. So the optimal incentive-compatible financial arrangement is just a standard risky debt contract, whose terms are the optimal solution to a standard principle-agent problem between entrepreneurs and banks.\textsuperscript{10}

Unlike the costly state verification framework similar to Carlstrom and Fuerst (1998), credit contracts are here nominal. They stipulate one-period loans, established after all aggregate shocks have occurred. Then, after the wholesale-good production takes place, each entrepreneur draws an idiosyncratic shock, $\omega_{it}$, a productivity and management efficiency shock, reflecting its management skills, recruitment efficiency, hires’ quality and input utilization skills, which is the source of wholesale firms’ heterogeneity. $\omega_{it}$ is i.i.d. with a continuous distribution function, $\Phi(\cdot)$, and a density function, $\phi(\cdot)$, defined over a non-negative support, and has a mean of unity and $\Phi(0) = 0$. Moreover, its variance, reflecting the shock’s volatility and the entrepreneurs’ riskiness, is time-varying and its standard deviation, $\sigma^\omega_t$, follows a first-order autoregressive process given by:

$$\log(\sigma^\omega_t) = (1 - \rho_\sigma) \log(\sigma^\omega_{t-1}) + \rho_\sigma \log(\sigma^\omega_{t-1}) + u^\sigma_t, \quad \rho_\sigma \in [0, 1)$$

where $u^\sigma_t \sim i.d. \sim N(0, \sigma^2_\sigma)$

$\sigma^\omega$ is the steady-state value of the standard deviation, $\sigma^\omega_t$.

Then, to justify the recourse to the external funding, entrepreneurs are assumed to pay their real production bill, $r^K_{it} K_{it} + W_{it} N_{it}$, as well as the real vacancies posting costs, $\gamma V_{it}$, prior to production. But due to idiosyncratic shocks, firms face default risk on their debt. For low values of $\omega_{it}$, some firms may not be able to reimburse the credit. Let $B_{it}$ be the total expected real amount of the production bill for a firm $i$, so that:

$$B_{it} = W_{it} N_{it} + r^K_{it} K_{it} + \gamma V_{it}$$

The wholesale firm borrows an amount of $P_t(B_{it} - X_{it})$ from the bank at a implicit interest rate, $R^l_t$, where $X_{it}$ is its real net worth. An entrepreneurs and a representative bank agree on a financial contract specifying a break-even entrepreneur-specific productivity level, $\bar{\omega}_{it}$, satisfying $R^l_t P_t(B_{it} - X_{it}) = \bar{\omega}_{it}(P^ws_t Y^ws_t + Z_t \psi_{it})$, where $Z_t$ is the total value of a new worker.

If $\omega_{it} < \bar{\omega}_{it}$, the firm is insolvent and the bank confiscates the total output produced. The bank can observe this state of nature at a monitoring cost $\mu_t \in (0, 1)$, a fraction of the realized value of the firm. After the

\textsuperscript{10}See Townsend (1979) and Gale and Hellwig (1985).
realization of shocks and production occurrence, this firm’s value is given by \( \omega_t (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) \). The monitoring is non-stochastic and the lender actions are pre-committed.

If \( \omega_t \geq \bar{\omega}_it \), the entrepreneur pays back the value \( \bar{\omega}_it (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) \), the loan amount augmented with interest. So, this framework breaks down the Modigliani-Miller theorem and makes the firms’ external borrowing costs higher than internal funds opportunity costs. Indeed, firms must borrow at a premium over the risk-free rate as seen below.

The expected return earned by the firm \( i \), \( E^f_{it} \), is given by:

\[
E^f_{it} = (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) \int_{\omega > \bar{\omega}_it} (\omega - \bar{\omega}_it) \phi(\omega) d\omega
\]  \hspace{1cm} (22)

Using the statistic properties of the random idiosyncratic shock, equation (22) can be developed to see that the expected return of the firm is a fraction of its total realized value:

\[
E^f_{it} = (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) f(\bar{\omega}_it)
\]  \hspace{1cm} (23)

where \( f(\bar{\omega}_it) = \int_{\omega > \bar{\omega}_it} \omega \phi(\omega) d\omega - \bar{\omega}_it [1 - \Phi(\bar{\omega}_it)] \). Note that \( f(\bar{\omega}_it) \in (0, 1) \) and \( f'(\bar{\omega}_it) = \Phi(\bar{\omega}_it) - 1 \leq 0 \).

The firm’s expected return is a decreasing function of \( \bar{\omega}_it \). This result is quite intuitive since an increase of the default rate \( \Phi(\bar{\omega}_it) \) reduces the gross share of return going to the firm.

Similarly, the expected return earned by the bank, \( E^b_{it} \), is given by:

\[
E^b_{it} = (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) \left[ \int_{\omega > \bar{\omega}_it} \bar{\omega}_it \phi(\omega) d\omega + (1 - \mu_t) \int_{\omega < \bar{\omega}_it} \omega \phi(\omega) d\omega \right]
\]

It is straightforward to show that lender’s expected return is also a fraction \( g(\bar{\omega}_it) \in (0, 1) \), of the total return of the wholesale firm. Then,

\[
E^b_{it} = (P_t^{ws} Y_{it}^{ws} + Z_t \psi_{it}) g(\bar{\omega}_it)
\]  \hspace{1cm} (24)

\[f'(\bar{\omega}) \leq 0, \forall \bar{\omega} \in [0, \infty). \] In addition, \( \lim_{\bar{\omega} \to 0} f(\bar{\omega}) = 1 \) and \( \lim_{\bar{\omega} \to \infty} f(\bar{\omega}) = 0 \). Then, \( f(\bar{\omega}) \in (0, 1) \).

\[1 - f(\bar{\omega}) \in (0, 1) \] and by definition, we have \( \int_{\omega < \bar{\omega}_it} \omega \phi(\omega) d\omega \in (0, 1) \) and \( \int_{\omega > \bar{\omega}_it} \bar{\omega}_it \phi(\omega) d\omega \in (0, 1) \), then \( g(\bar{\omega}) \in (0, 1) \), \( \lim_{\bar{\omega} \to 0} g(\bar{\omega}) = 0 \) and \( \lim_{\bar{\omega} \to \infty} g(\bar{\omega}) = 1 - \mu. \)
where,
\[ g(\bar{\omega}_{it}) = 1 - f(\bar{\omega}_{it}) - \mu_t \Gamma(\bar{\omega}_{it}) \]

with \( \Gamma(\bar{\omega}_{it}) = \int_{\omega < \bar{\omega}_{it}} \omega \phi(\omega) d\omega \). An amount of the realized firm’s value, \( \mu_t \Gamma(\bar{\omega}_{it}) \), is lost due to monitoring in cases of declaring bankruptcy by the borrower. In this context, the monitoring cost spending is a synonym of bankruptcy and it is spent in terms of currency so that bankruptcy has no impact on the real output. As a consequence, the proportion of the total amount recovered by the bank in case of bankruptcy is \( (1 - \mu_t) \Gamma(\bar{\omega}_{it}) \). \( (1 - \mu_t) \) is interpreted as the recovery rate. Following Livdan et al. (2009) and Petrosky-Nadeau (2014), this recovery rate is assumed to be time-varying and to have the following specification:

\[ 1 - \mu_t = s_{0,t} \exp s_1(\omega_{it} - 1) \]

(25)

\( s_1 \) is the elasticity of the recovery rate to the entrepreneurial productivity level and \( s_{0,t} \) is interpreted as a credit shock, following a first-order auto-regressive process:

\[ \log s_{0,t} = (1 - \rho_{s0}) \log s_0 + \rho_{s0} \log s_{0,t-1} + u_{s0}^0, \quad \rho_{s0} \in [0, 1) \text{ where } u_{s0}^0 \ iid \sim N(0, \sigma_{s0}^2) \]

where \( s_0 \) is the steady-state value of the credit shock, \( s_{0,t} \).

Finally, the optimal contract is a solution to a maximization problem, where the entrepreneur maximizes its expected return subject to the bank’s participation constraint (Paustian (2004)). The bank is willing to lend funds only and only if the contract yields an expected return greater or equal to the riskless rate of return, \( R_t \):

\[
\max_{K_{it}, H_{it}, V_{it}, \omega_{it}} \quad [P_t^{ws}Y_{it} + Z_t \psi_{it}] f(\bar{\omega}_{it}) \\
\text{subject to} \quad [P_t^{ws}Y_{it} + Z_t \psi_{it}] g(\bar{\omega}_{it}) \geq R_t P_t (W_t N_{it} + \gamma V_{it} + r_t K_{it} - X_{it}) \\
\psi_{it} = p_t V_{it} \\
N_{it} = n_{it} H_{it}
\]

(26)
The first-order conditions for the firm’s problem are summarized by the three following equations:

\[ Y_K(K_{it}, H_{it} | n_{it}) = \frac{P_t}{P_{ws}} r^K_t R_t S_{it} \]  \hspace{1cm} (27)

\[ Y_H(K_{it}, H_{it} | n_{it}) = \frac{P_t}{P_{ws}} n_{it} W_t R_t S_{it} \]  \hspace{1cm} (28)

\[ Z_t = \gamma \frac{R_t S_{it}}{P_t} \]  \hspace{1cm} (29)

where \( S_{it} = \{1 - \mu_t [\Gamma(\bar{\omega}_{it}) + \bar{\omega}_{it} h(\bar{\omega}_{it}) f(\bar{\omega}_{it})]\}^{-1} \), with \( h(\bar{\omega}_{it}) \) the hazard rate\(^{13}\) defined by \( h(\bar{\omega}_{it}) = \frac{\phi(\bar{\omega}_{it})}{1 - \Phi(\bar{\omega}_{it})} \).

Equations (27) and (28) show that marginal products of labor is equal for all firms, so that the capital-labor ratio is constant across firms because of the linearity assumption on the monitoring technology and the homogeneity of the Cobb-Douglas production function. Therefore, \( S_{it} \) does not depend on \( i \) and the threshold value of the entrepreneurial productivity, \( \bar{\omega}_{it} \), is identical for all firms. Consequently, the subscript \( i \) can be dropped in what follows. So, the assets’ distribution among entrepreneurs does not matter for the equilibrium\(^{14}\).

Further, asymmetric information in the credit market generates inefficiencies in both markets: the wholesale-good market and the labor market. On the one hand, the marginal productivity of labor is higher than its corresponding real marginal costs. The final real price of the wholesale good is augmented by a financial mark-up, \( S_t > 1 \)\(^{15}\), used to overcome the agency problem between entrepreneurs and banks. As a consequence, credit market conditions matter because they affect firms’ marginal costs and are transmitted to the rest of the economy through the selling price’s mark-up. Banks have a margin behavior, that will pass trough the rest of the economy by the wholesale-good price. Aggregating over entrepreneurs and embedding equations (27)-(29) in the lender’s break even constraint, the firms’ leverage ratio, \( \mathcal{L}_t \), is given by:

\[ \mathcal{L}_t \equiv \frac{B_t}{X_t} = \frac{1}{1 - S_t(\bar{\omega}_t) g(\bar{\omega}_t)} \]  \hspace{1cm} (30)

with, \( \mathcal{L}_\bar{\omega} > 0 \). For a given level of net worth \( X_t \), a higher leverage ratio is associated with a high default rate. The probability of default increases as the loan amount raises (see Stiglitz and Weiss (1981)).

\(^{13}\)We assume that \( \bar{\omega} h(\bar{\omega}) \) is increasing in \( \bar{\omega} \) in order to ensure the concavity of the lender’s net share of return, \( g(\bar{\omega}) \), and avoid any credit rationing at the equilibrium. This regularity condition is without loss of generality and it is satisfied by most of the continuous probability distributions. See Bernanke et al. (1999) for details.

\(^{14}\)The ex-post heterogeneity among entrepreneurs is introduced by the level of their idiosyncratic productivity at each period, which makes their wealth fluctuating over time. Thanks to the linearity assumptions, the aggregation over entrepreneurs is possible and complications stemming from heterogeneity are ruled out.

\(^{15}\)See appendix A for the proof.
It is straightforward to show that, at the optimum, equation (26) is binding. Using this result with the condition on $\bar{\omega}_t$, the risk premium, $\Delta_t$, defined as the ratio of the lending rate to the risk free rate, $R^l_t/R_t$, is given by:

$$\Delta_t = \frac{\bar{\omega}_t}{g(\bar{\omega}_t)}$$

Figure 3. shows the evolution of the risk premium as a convex increasing function of the entrepreneurial productivity threshold, $\bar{\omega}$. This result is quite familiar in the financial accelerator literature. A higher default probability of firms induces a higher cost of lending for banks and consequently, a higher loan spread. In that case, an expansionary monetary policy for example affects the loan rate by decreasing the opportunity cost of lending funds for banks. The loan rate decreases more than one-to-one with respect to the risk-free rate; firms are able to repay their debts more easily and the default probability must decrease. As a result, in equilibrium, credit spreads, average financial distortion and the mark-up must fall.

On the other hand, the real value of a new hire depends also on the financial contract conditions. Besides the unit cost stemming from hiring, $\gamma$, and the average duration of vacancies, $1/p_t$, the total cost of vacancy posting is augmented by the same financial mark-up. Financial contract conditions affect the labor market efficiency through the total vacancy posting cost, that becomes an endogenous variable. This relation is presented by figure 4., where the real posting cost is also an increasing and convex function of $\bar{\omega}$ and its slope raises with monitoring costs, $\mu$. For a higher default likelihood (higher $\bar{\omega}$), banks charge a higher risk premium, $\Delta$, so that entrepreneurs obtain their credit at a higher lending rate, $R^l_t$, which makes their external funds more expensive and reduces their willingness to open vacancies.

An increase of $\mu$ shifts the real vacancy posting cost upward. For a fixed level of $\bar{\omega}$, the real cost of a new hire raises with monitoring costs. As these costs are expressed in terms of currency, and not in terms of physical goods, they do not generate a loss of resources through a destruction of goods, which could have been used for consumption, but they generate an additional cost taken into account by banks when agreeing on an appropriate interest rate on loans. Fluctuations in monitoring costs and bankruptcy rates will have an impact on welfare only indirectly, through their implications on the mark-up.

Now, by making use of the optimal financial contract conditions derived below and the hiring condition
given by equation (21), the job creation condition under financial frictions in the credit market is obtained:

$$\frac{\gamma}{p_t} R_t S_t = W_t H_t [R_t S_t - 1] + (1 - \delta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{\gamma}{p_{t+1}} R_{t+1} S_{t+1}$$

(31)

For any positive cost of monitoring, financial frictions increase as expected the average cost of filling a vacancy. The evolution of credit market conditions changes the opportunity cost for resources firms use to create new jobs in the face of small changes in the expected benefit to a new worker. It alters the dynamics of job vacancies. So credit spreads are a key element to understand the cyclical behavior of job creation and the dynamics of labor markets. Agency problems on credit markets affect the performance of labor markets. And labor market conditions will be an alternative channel for transmission of monetary policy shocks that affect the cost of credit.

3.3.3 Wage bargaining

Bellman equations

The real hourly wage is assumed to be determined on a period-by-period basis and through a Nash bargaining between a representative entrepreneur and a representative household. The Nash real hourly wage splits the joint surplus of the employment relationship between them, depending on their respective bargaining power. As in Andolfatto (1996), the representative household chooses for all its member the
labor supply. We assume that each household bargains with each employer separately by taking as given wages in all other matches.

Given that the worker’s welfare is independent of the number of employed persons by the firm at which he or she is working, all workers have the same welfare (and thus the same surplus). So that, the discounted value of employment for a worker in terms of current consumption at time \( t \) is denoted by \( W_t^N \) and given by:

\[
W_t^N = W_t H_t - \frac{\varepsilon_t H_t^{1+\tau}}{(1 + \tau)\lambda_t} + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \delta_t(1 - q_{t+1})) W_{t+1}^N + \delta_t(1 - q_{t+1}) W_{t+1}^U\right] \quad (32)
\]

The discounted value of a job for a worker in terms of current consumption is the sum of the total real wage earned, reduced for the marginal disutility of working and the expected discounted gain from being either employed or unemployed during the subsequent period. A worker will be again employed at the period \( t + 1 \) if the match has not been destroyed, with a probability \( (1 - \delta_t) \), after the matching of the period \( t \); or if the match has been destroyed with the probability \( \delta_t \) after the matching of the period \( t \), but that another matching occurs at the period \( t + 1 \) with the probability \( q_{t+1} \). And a worker will become unemployed at the period \( t + 1 \) if the match is destroyed after the matching of the period \( t \) and if he or she does not find a job at the period \( t + 1 \), that to say with the probability \( \delta_t(1 - q_{t+1}) \).

The discounted value of unemployment for a worker in terms of current consumption at time \( t \) is denoted by \( W_t^U \) and given by:

\[
W_t^U = b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1} W_{t+1}^N + (1 - q_{t+1}) W_{t+1}^U\right] \quad (33)
\]

In the same spirit, the discounted value to be unemployed for a worker in terms of current consumption is the sum of real unemployment benefits and the expected discounted gain from either being employed (with probability \( q_{t+1} \)) or unemployed (with probability \( 1 - q_{t+1} \)) after the matching of the period \( t + 1 \). Therefore, the worker’s surplus of an employment relationship is given by:

\[
W_t^N - W_t^U = W_t H_t - \frac{\varepsilon_t H_t^{1+\tau}}{(1 + \tau)\lambda_t} - b + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[(1 - \delta_t)(1 - q_{t+1})(W_{t+1}^N - W_{t+1}^U)\right] \quad (34)
\]

For a firm, the discounted value of an employed worker at time \( t \) in terms of current consumption is defined
as:

$$J_{n,t} = \frac{P_{t}^{ws}}{P_{t}} (1 - \alpha) Y_{t}^{ws} - W_{t} H_{t} + \beta E_{t} \frac{\lambda_{t+1}}{\lambda_{t}} (1 - \delta_{t}) J_{n,t+1}$$  \hspace{1cm} (35)$$

This discounted value is equal to the current profits from an employed worker, plus the expected discounted continuation value. The job is still provided at the period $t + 1$ if the match has not been destroyed with a probability $(1 - \delta_{t})$ after the matching of the period $t$. And if the job is not provided at the period $t + 1$, the continuation value will be equal to zero.

Then, the discounted value of an open vacancy for a firm in terms of current consumption at time $t$ is given by:

$$V_{t} = -\frac{Z_{t}}{P_{t}} + \frac{\beta E_{t}}{\lambda_{t}} \left[ p_{t+1} J_{n,t+1} + (1 - p_{t+1}) V_{t+1} \right]$$  \hspace{1cm} (36)$$

The discounted value of an open vacancy for a firm in terms of current consumption is equal to the real total hiring costs of the vacancy, plus the expected discounted gain to fill or not the vacancy during the following period. The vacancy will be filled at the period $t + 1$ with a probability $p_{t+1}$, if a match occurs during the matching of this period. Otherwise, with a probability $(1-p_{t+1})$, the job remains vacant in $t+1$.

Nash bargaining and wage setting

The Nash wage bargaining consists of maximizing the net surplus of the employment relationship for a representative firm and household’s pair, depending on the households’ bargaining power, $\eta_{t}$. This latter is assumed to be time-varying and to follow an auto-regressive process given by:

$$\log(\eta_{t}) = (1 - \rho_{\eta}) \log(\eta) + \rho_{\eta} \log(\eta_{t-1}) + u_{\eta}^{\eta}, \quad \rho_{\eta} \in [0, 1) \text{ where } u_{\eta}^{\eta} \overset{iid}{\sim} N(0, \sigma^{2}_{\eta})$$

As a consequence, the chosen real hourly wage is the one that maximizes:

$$\max_{W_{t}} \left( W_{t}^{N} - W_{t}^{U} \right)^{\eta} \left( J_{n,t} - V_{t} \right)^{(1-\eta)}$$

where $(W_{t}^{N} - W_{t}^{U})$ is the net surplus of households (expressed in 34) and $(J_{n,t} - V_{t})$ is the net surplus of
firms, for an employment relationship.

Since there is a free entry, at the equilibrium, the vacancy posting condition, $V_t = 0$, is satisfied. Indeed, if $V_t > 0$, a firm has an incentive to post vacancies as the value of a vacant job is positive. As the number of vacancies increases relative to the number of unemployed workers, the probability to fill a vacancy, $p_t$, decreases. Indeed, the labor market tightness, $\theta_t$, increases and $p_t$ decreases since $\partial p_t/\partial \theta_t < 0$. That reduces the incentive to post vacancy jobs and diminishes the value of $V_t$, until it equals zero. Therefore, the first-order necessary condition for the Nash bargaining solution is given by:

$$\eta_t J_{n,t} = (1 - \eta_t)(W_{t}^N - W_{t}^U)$$

The following Nash real hourly wage is obtained:

$$W_t H_t = \frac{P^{ws}_t (1 - \alpha) Y^{ws}_t}{\pi_t} + (1 - \eta_t) \left[ b + H^1 (1 + \tau) \theta_t \right] + (1 - \eta_t) \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \delta_t) p_{t+1} \theta_{t+1} (W_{t+1}^N - W_{t+1}^U)$$

The wage shares costs and benefits from the match between workers and firms according to the parameter $\eta_t$. Workers obtain a fraction $\eta_t$ of the firm’s revenues and are compensated for a fraction $(1 - \eta_t)$ for the disutility they suffer from supplying hours of work and for the foregone unemployment benefits (workers’ outside opportunities). A new element is the expected labor market tightness. If a matching is broken, workers and entrepreneurs have to look for another partners in next periods, which is costly. This cost is thus incorporated in the wage. Rearranging equation (38) by taking into consideration the equilibrium prices given by equations (27) and (28), the following new wage equation is obtained:

$$W_t H_t = \frac{1 - \eta_t}{1 - \eta_t R_t S_t} \left[ b + H^1 (1 + \tau) \theta_t \right] + \frac{1 - \eta_t}{1 - \eta_t R_t S_t} \left[ \beta E_t \frac{\lambda_{t+1}}{\lambda_t} (1 - \delta_t) p_{t+1} \theta_{t+1} (W_{t+1}^N - W_{t+1}^U) \right]$$

As a consequence, asymmetric information in financial market altered significantly the real bargaining power of firms and workers.

### 3.4 Intermediate-good production

There is a continuum of monopolistically competitive retailers indexed by $j \in [0, 1]$. They buy from entrepreneurs homogeneous goods at the price $P^{ws}_t$. They differentiate costlessly each unit of these goods into a unit of retail goods, $Y_{j,t}$. These firms are assumed to have no other inputs or costs than the
homogeneous good. Following Yun (1996), Christiano et al. (2005) and Trigari (2009), a price stickiness
is formulated in the spirit of Calvo (1983). Every period, only a random fraction \( \varrho \in [0, 1) \) of firms, is
able to fully re-optimize their nominal prices knowing the aggregate shock, \( A_t \). This hazard rate, \( \varrho \), is
constant across firms and time. The remaining fraction of firms does not re-optimize their prices and
following Christiano et al. (2013), they keep their prices unchanged. So the price set by a retailer \( j \), \( P_{j,t} \),
corresponds to:

\[
P_{j,t} = \begin{cases} 
  P_{j,t-1} & \text{with probability } 1 - \varrho \\
  \tilde{P}_t & \text{with probability } \varrho 
\end{cases} 
\]  

(40)

where \( \tilde{P}_t \) is the optimal price set by the fraction \( \varrho \) of retailers who are able to re-optimize their prices.
Note that there is no price indexation, to replicate the observation that many prices can remain unchanged
over time (Eichenbaum, Jaimovich and Rebelo, 2011 and Klenow and Malin, 2011).

Now, the price index of the intermediate goods, \( P^R_t \), is given by:

\[
P^R_t = \left[ \int_0^1 P^{1-\epsilon_t}_{j,t} \, dj \right]^{\frac{1}{1-\epsilon_t}} = \left[ \varrho (\tilde{P}_t)^{1-\epsilon_t} + (1 - \varrho) (P_{t-1})^{1-\epsilon_t} \right]^{\frac{1}{1-\epsilon_t}} 
\]  

(41)

where \( \epsilon_t > 1 \) is a time-varying parameter governing the degree of monopolistic competition (or the time-
varying elasticity of substitution between intermediate goods in final-good production). It follows an
exogenous first-order auto-regressive process:

\[
\log(\epsilon_t) = (1 - \rho_\epsilon) \log(\epsilon) + \rho_\epsilon \log(\epsilon_{t-1}) + u_\epsilon^t, \quad \rho_\epsilon \in [0, 1) \text{ where } u_\epsilon^t \sim iid N(0, \sigma_\epsilon^2) 
\]  

(42)

where \( \epsilon \) is the steady-state value for the degree of monopolistic competition, \( \epsilon_t \).
The price index is a CES aggregate of all retail goods prices in the economy at \( t \). A costless price regulation
mechanism is assumed, which guarantees that a consumer pays the same price whatever the firm at which
he realizes his purchases\(^{16}\). So, the uniform price index corresponds to a weighted average price of the
fraction \( \varrho \) of firms who can re-optimise their prices after the aggregate shock, \( A_t \), and the fraction \( (1 - \varrho) \)
of firms who can not. The sum in equation (41) can be transformed into a convex combination of two
prices because firms of each type are respectively symmetric.

Firms that can re-optimise their price, maximize the expected discounted value of their profits, since
firms expect to keep this price for more than the current period. If the expected probability of price

\(^{16}\)The matching of consumers and firms is ignored.
stickiness is high, firms able to re-optimize their price at the period $t$ will be relatively more concerned about the future when they make their current pricing decisions.

Thus, these firms face the following problem:

$$\max_P E_t \sum_{s=0}^{\infty} (\rho \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left[ \left( \frac{\tilde{P}_t}{P_{t+s}} \right)^{-\epsilon_{t+s}} Y_{t+s}^{fl} \left( \frac{\tilde{P}_t}{P_{t+s}} - \frac{P_{WS}}{P_{t+s}} \right) \right]$$

where $Y_{t}^{fl}$ is the production of a representative flexible-price firm.

The first-order condition of the problem is given by:

$$E_t \sum_{t=0}^{\infty} (\rho \beta)^s \frac{\lambda_{t+s}}{\lambda_t} \left( (\epsilon_{t+s} - 1)P_{WS}^{-1-\epsilon_{t+s}} P_{t+s}^{(1-\epsilon_{t+s})} - \tilde{P}_t^{(-\epsilon_{t+s}-1)} Y_{t+s}^{fl} \right) = 0$$

(43)

The flexible price set by firms who are able to re-optimize their prices is given by:

$$P_t^* = \frac{\epsilon_{t+s}}{\epsilon_{t+s} - 1} P_{t+s}^{\text{ws}}, \quad \text{where} \quad \frac{\epsilon_{t+s}}{\epsilon_{t+s} - 1} > 1$$

(44)

Flexible-price firms set their price such that it equals the present discounted value of marginal costs. The size of the markup depends on the elasticity of the demand to the price. If there is no price-stickiness, $\varrho = 0$, the monopoly standard mark-up formula is obtained:

$$\tilde{P}_t = \frac{\epsilon_{t+s}}{\epsilon_{t+s} - 1} P_{t+s}^{\text{ws}}, \quad \text{where} \quad \frac{\epsilon_{t+s}}{\epsilon_{t+s} - 1} > 1$$

(45)

Finally, the wholesale-goods market clearing condition suggests that the total output, after realization of the entrepreneurial idiosyncratic shock, $\omega_t$, has to be absorbed by fixed-price and flexible-price firms’ input demands, respectively:

$$\omega_t Y_{t}^{ws} = \varrho Y_{t}^{fl} + (1 - \varrho) Y_{t}^{fx}$$

(46)

where $Y_{t}^{fx}$ is the production of firms not able to reset their prices.
3.5 Final-good production

Final-good firms are the last one to sell in the economy. They proceed in a perfectly competitive market and are owned by households. They purchase a continuum of differentiated intermediate goods, $Y_{j,t}$, and aggregate these varieties to produce $Y^f_t$ units of the final good. They have no other cost, except the one to buy to intermediate-good firms the different varieties. The final good will then be sold to households (to consume and accumulate capital) and to solvent exiting entrepreneurs, at a price $P_t$, the consumer’s price index.

Given that retailers are respectively symmetric, final goods are produced using a standard constant return to scale technology given by:

$$Y^f_t = \left[ \int_0^1 Y^e_{j,t}^{\epsilon_{t-1}} dj \right]^{\epsilon_t/\epsilon_{t-1}} = \left[ \varrho \left[ Y^f_t \right]^{\epsilon_t/\epsilon_{t-1}} + (1-\varrho) \left[ Y^e_t \right]^{\epsilon_t/\epsilon_{t-1}} \right]^{\epsilon_t/\epsilon_{t-1}} \quad (47)$$

The quantity of final goods produced is equal to a weighted average sum of the intermediate goods quantity.

Each competitive final-good firms choose their own input demand functions for each variety of intermediate goods, $Y_{j,t}$, so as to maximize their nominal profit, $\Pi^f_t$, defined as:

$$\Pi^f_t = P_t Y^f_t - \int_0^1 P_{j,t} Y_{j,t} dj$$

where $P_t$ is the bundler’s technology price-index that corresponds to the consumer’s price index and $P_t(j)$ is the input price. The solution to the maximization problem\(^\dagger\) yields the following demand function for the intermediate good of variety $j$:

$$Y_{j,t} = \left[ \frac{P_{j,t}}{P_t} \right]^{-\epsilon_t} Y^f_t \quad (48)$$

So, the demand for each variety of intermediate good is a downward sloping demand curve, which gives to the intermediate-good firms some pricing power, as seen before. According to equation (44), the mark-up depends negatively on the time-varying elasticity of substitution, $\epsilon_t$. The higher the elasticity, the lower the mark-up is and the higher is the demanded quantity.

\(^\dagger\)Final-good firms maximize their expected stream of profits, which is equivalent to maximizing their profit period by period since they purchase intermediate goods at the same frequency.
Then, as we are in a competitive setting, the zero-profit condition applies at the equilibrium and it yields:

\[ P_t Y^f_t = \int_{j=0}^{1} P_{j,t} Y_{j,t} dj \]

From this condition, the output price or the consumer’s price index can be easily derived by plugging the demand function into the zero-profit condition:

\[ P_t Y^f_t = \int_{j=0}^{1} P_{j,t} \left( \frac{P_{j,t}}{P_t} \right)^{\epsilon_t} Y^f_t dj \]

which gives:

\[ P_t = \left[ \int_{j=0}^{1} P_{j,t}^{(1-\epsilon-t)} dj \right]^{\frac{1}{1-\epsilon_t}} \] (49)

The final-good market clearing condition implies that final goods, \( Y^f_t \), may be consumed by households, \( C_t \), and entrepreneurs, \( C^e_t \), or be used as investment, \( I_t \), as government expenditures, \( G_t \) or as capital utilization costs, \( \Upsilon(\nu_t)K^p_{t-1} \). The following aggregate resource constraint is thus obtained:

\[ Y^f_t = C_t + C^e_t + I_t + G_t + \Upsilon(\nu_t)K^p_{t-1} \] (50)

### 3.6 Entrepreneurs

Entrepreneurs own wholesale-good firms. At the beginning of period \( t \), they borrow from banks to cover hiring costs, wage bill and renting capital because they have to pay them prior to the production. Then, after receiving the amount borrowed from the bank, they rent and pay immediately capital from households on a perfectly competitive capital market. At the same time, vacancies are posting and matches take place with workers. Entrepreneurs pay also immediately vacancy posting costs and wages of workers hired and of workers already hired at previous periods. Finally, after the wholesale-good sale, some entrepreneurs will be declared solvent or bankrupt, depending on their production and idiosyncratic shock levels. Entrepreneurs declared bankrupt are not able to reimburse their entire loan. The bank will confiscate the proceeds of the production. As a consequence, bankrupted entrepreneurs will not consume, nor accumulate net-worth. But at the beginning of the next period \( t+1 \), entrepreneurs are assumed to receive an exogenous endowment, which will be used as collateral or net worth to borrow from banks at the beginning of the next period, in order to be sure that this kind of agents, specialized in managing production, will be able to continue their activities. Entrepreneurs declared solvent reimburse their entire loan to
banks. Then, they have to wait the production of the final good to consume or accumulate a net worth.
To ensure that entrepreneurs do not accumulate net worth, such that they could be able to self-finance their production at the next period, we assume that each entrepreneur has a constant probability, $\varsigma$, to die at the end of the period. It will limit the size of aggregate net worth in an infinite horizon set up. Indeed, since the rate of return on internal funds is higher than the one of external funds, due to asymmetric information on credit markets, risk neutral entrepreneurs may be willing to postpone consumption and would only accumulate funds.

So solvent entrepreneurs who exit the economy at the end of the period will consume all their net worth. Thus, the aggregate entrepreneurial consumption, $C_e^t$, is given by:

$$C_e^t = (1 - \varsigma) \frac{P^{ws}}{P_t} Y^{ws} f(\bar{\omega}_t)$$  \hspace{1cm} (51)

To be sure however to have a constant fraction of entrepreneurs in the economy in every period, we assume that the birth of rate of entrepreneurs at the beginning of each period ensures this constant fraction.

Solvent entrepreneurs who do not exit the economy at the end of the period will keep accumulating net worth using their realized return. Consequently, the evolution of the aggregate entrepreneurial real net worth is given by:

$$X_{t+1} = \varsigma \frac{P^{ws}}{P_t} Y^{ws} f(\bar{\omega}_t)$$  \hspace{1cm} (52)

### 3.7 Monetary and fiscal policy

**Monetary policy**

The monetary policy is decided and carried out by the central bank at the beginning of the period $t$, following an interest rate Taylor-type rule\textsuperscript{18}. The nominal interest rate of each period will be set depending on deviations in output growth, inflation and nominal interest rate from their steady-state level:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_R} \left[\frac{\pi_t}{\pi}\right]^{\rho_{\pi}} \left(\frac{Y_t}{Y}\right)^{\rho_Y} \gamma^{1-\rho_R} \epsilon_t R$$  \hspace{1cm} (53)

\textsuperscript{18}The same kind of Taylor rule is used by Gertler et al. (1999), Krause et al. (2008) and Trigari (2009).
where $\rho_R$ is the degree of interest rate smoothing, $\rho_Y$ and $\rho_\pi$ are the response coefficients to output and inflation variables and variables without a time subscript are steady state values. $\epsilon_t^R$ is the monetary policy shock, following a first-order auto-regressive process:

$$\log(\epsilon_t^R) = \rho_R \log(\epsilon_{t-1}^R) + u_t^R, \quad \rho_R \in [0,1)$$

where $u_t^R \sim N(0, \sigma_R^2)$

**Fiscal policy**

The fiscal policy is decided and carried out by the government at the end of the period $t$. After the final-good production, households pay nominal lump-sum taxes (or subsidies), $T_t$, to the government. With these taxes, the government will be able to finance real exogenous government spending, $G_t$, and the amount of real unemployment benefits, $(1-n_t)b_t$, for unemployed workers. The level of the unemployed is known by the government at the end of the matching process on the labor market. So the government budget constraint is the following:

$$G_t + (1-n_t)b = \frac{T_t}{P_t}$$ (54)

where $G_t$ obeys to:

$$G_t = \left(1 - \frac{1}{\epsilon_t^G}\right)Y_t^f$$ (55)

Following Gertler et al. (2008), the government spending shock, $\epsilon_t^G$, follows a first-order auto-regressive process given by:

$$\log(\epsilon_t^G) = (1 - \rho_G) \log(\epsilon_{t-1}^G) + \rho_G \log(\epsilon_{t}^G) + u_t^G, \quad \rho_G \in [0,1)$$

where $u_t^G \sim N(0, \sigma_G^2)$

where $\epsilon_t^G$ is the long-run target level of government spending, $\epsilon_t^G$.

**4 A structural Bayesian VAR approach**

Empirical evidence are presented by using a structural Bayesian VAR (SBVAR). A 10 variable SBVAR using quarterly data for the United States (US) between 1970Q1-2005Q1 is applied. We estimate the dynamic response of labor and credit markets variables to identified exogenous monetary policy and credit
shocks.

The SVAR approach is assumed to infer an underlying economic theory. It is a good alternative to
the traditional a-theoretic VAR, since economic theory plays a key role in the modeling identifying pro-
cess. Moreover, we use Bayesian methods to estimate the VAR model (Stock and Watson (2001)). The
difference with VAR models is that model parameters are treated as random variables, and priors are
assigned to them. Indeed, with VAR models, the parameter space increases with the number of dependent
variables and the number of lags. In our estimation, we have a consequent number of dependent variables
and our quarterly database is only of moderate size. Thus, empirical Bayes estimators enable to overcome
the over-fitting problem of standard VAR models. The inclusion of prior information enables better pre-
cision of forecasts from VAR models and reinforces the link between VAR analysis and the economic theory.

4.1 Data and identification strategy

4.1.1 Data

A structural VAR is a \( p \)-equations, \( p \)-variables model in which each variable is explained by its lagged
values, current and past values of the other variables. So the model aims to capture linear interdependences
among multiple variables. The structural VAR considered is given by:

\[
A_0 X_t = a_0 + A_1 X_{t-1} + A_2 X_{t-2} + ... + A_L X_{t-L} + \epsilon_t, t = 1, ..., T
\] (56)

where \( X_t \) is a \( p \times 1 \) vector of variables at date \( t \), \( a_0 \) is a \( p \times 1 \) vector of constants and \( A_i \) is a \( p \times p \) coefficient
matrix for each lag of the variable vector. The main diagonal terms of the \( A_0 \) matrix is normalized to
a \( 10 \times 1 \) vector of ones. \( \epsilon_t \) is \( p \times 1 \) structural shocks vector, associated to the variable vector, with a
variance-covariance matrix, \( \Sigma \). We assume that \( E(\epsilon_t \epsilon_t') = \Sigma, E(\epsilon_t) = 0 \) and \( E(\epsilon_t \epsilon_s') = 0, \forall t \neq s \). So, error
terms have a zero mean. Structural shocks are supposed to be independent. There is no serial correlation
in individual error terms, so \( \Sigma \) is a diagonal matrix. If they were correlated, this would suggest some
remaining unexplained causal relationship between structural shocks, and a failure in the identification
process.

In our estimation, \( p \) is equal to 10 variables and we use quarterly data. The vector \( X_t \) is composed of
three elements:

\[
X_t \equiv [z_t \ R_t \ creditspreads_t]
\]
\[ z_t = [\log(Y_t) \log(\pi_t) \log(J_{crea}) \log(J_{destr}) \log(W_t) U_t \log(H_t) D_t]' \]

The vector of variables, \( X_t \), is partitioned in three components. In the first component, \( z_t \), the variables are real output, \( Y_t \), inflation, \( \pi_t \), to which we add five labor market variables and one credit market variable. The labor market variables are unemployment, \( U_t \), hours worked per employee, \( H_t \), real wages, \( W_t \), job creation and destruction rate, \( J_{crea} \) and \( J_{destr} \). The credit market variable is the default rate, \( D_t \). Then, there is the nominal interest rate, \( R_t \) and the last credit market variables are either the Baa-Aaa spread for the monetary policy shock or the Gilchrist-Zakrajsek spread for the credit shock.

The real output corresponds to the logarithm of real gross domestic product (GDP). The nominal interest rate is the Federal funds rate and inflation is measured as the change in logarithm of the GDP deflator between two consecutive quarters. Job-creation and job-destruction data are obtained from Davis et al. (2006) database. These data concern the manufacturing sector and are, respectively, the logarithm of job creation rate for start-ups and continuing establishments and the logarithm of job destruction rate for shutdowns and continuing establishments. Real wages are the logarithm of the real compensation per hour in the non-farm business sector. Unemployment is given by the ratio of civilian unemployed persons to the civilian labor force. Finally, hours worked per employee corresponds to the logarithm of the average weekly hours worked per employee of the non-farm business sector. The default rate is the default rate for Moody’s rated US speculative-grade corporate bonds, as used by Helbling et al. (2011) and Meeks (2012).

The Baa-Aaa spread is the Moody’s seasoned Baa-Aaa corporate bond yield. Finally, the Gilchrist-Zakrajsek spread (GZ spread) is a credit spread index constructed by Gilchrist and Zakrajsek (2012). This credit spread measures the difference in yields between a private debt instrument and government securities of comparable maturity. Formally, the authors have constructed the index as follow:

\[ S^{GZ}_t = \frac{1}{N_t} \sum_i \sum_k S_{it}(k) \]  \hspace{1cm} (57)

where \( N_t \) is the number of bond observations in quarter \( t \) and \( S_{it}(k) \) is the credit spread, expressed as the difference between the yield of the corporate bond \( k \) and a hypothetical Treasury security with exactly the same cash-flows as the underlying corporate bond. More precisely, for each individual bond issue in the sample, they construct a theoretical risk-free security that replicates exactly the promised cash-flows of the corresponding corporate debt instrument. So they compare the interest rate actually paid by the firm with what the US government would have paid on a loan with a similar maturity.\(^{19}\)

\(^{19}\)The GZ spread is an arithmetic average of the credit spreads on bonds in any given quarter. According to the authors, it has a large predictive power for the economic activity over the 1973-2010 period. It is based on micro level data set of
The VAR model is estimated in log levels, except for the interest rate, unemployment rate, default rate and credit spreads, which are included in percent. Lag lengths are determined for each shock by using Schwarz-Bayes (SB), Hannan-Quinn (HQ) and Akaike’s Final Prediction Error (FPE) criteria. The optimal number of lags is equal to \( L = 1 \).

Empirical evidence are presented for the United-States between 1970Q1-2005Q1. We decide to stop before the Great Recession, either before the Federal Reserve begins unconventional monetary policy measures. The identification strategy adopted is not able to capture exactly this type of measures. Furthermore, our data on job creation and job destruction are only available until 2005Q1.

The structural VAR model is used to study the impact of monetary and credit shocks on labor and credit market variables. Identification assumptions are required in order to interpret correlation as causality among variables. Both shocks identification is based on the recursiveness assumption as in Christiano et al. (2005) and Popescu and Smets (2010).

4.1.2 Monetary policy shocks identification

To identify monetary policy shocks, we use the recursiveness assumption as in Christiano et al. (1999) and Christiano et al. (2013).

First, the short-term nominal interest rate, the Federal funds rate \( R_t \), is considered as the instrument of monetary policy. The monetary authority is assumed to conduct its monetary policy following a simple reaction function, as follow:

\[
R_t = f(\Omega_t) + \epsilon^R_t
\]  

\( f \) is a linear function, \( \Omega_t \) is the information set of the monetary authority and \( \epsilon^R_t \) is the monetary policy shock.

The monetary policy shock, \( \epsilon^R_t \), is assumed to be orthogonal to the information set of the monetary authority, \( \Omega_t \). This information set includes the contemporaneous values of all variables included in \( z_t \) (eight variables), but the contemporaneous value of the Baa-Aaa credit spread is not included. This assumption means that no variable in \( z_t \) may respond contemporaneously to an unexpected change in the Federal funds rate, except for the Baa-Aaa spread. Thus, the dynamic responses to a monetary policy...
shock are identified by assuming that the monetary authority observes the current and lagged values of all variables in $z_t$, and only the lagged values of credit spreads. So, this process will depend on the Federal funds rate current realizations. So, the order of the variables included in the vector $X_t$ imposes a number of additional short-run restrictions, which corresponds to the Cholesky decomposition. However, the ordering of variables in $z_t$ does not influence the coefficients in the impulse response functions.

Our decision to include all variables except for the Baa-Aaa spread can be justified by the idea that some macroeconomic variables do not respond instantaneously to policy shocks. Friedman and Kuttner (1998), Gilchrist et al. (2009) and Gilchrist and Zakrajšek (2011) show empirically that credit spreads are monetary policy indicators, and have a strong information content with respect to business cycle fluctuations. They are as a mirror of monetary policy. Furthermore, even if the assumption can be criticized, our analysis is consistent as the same assumption is made in our theoretical model (Christiano et al. (2005)).

4.1.3 Credit shocks identification

Similar to Helbling et al. (2011), we want to see if exogenous shocks originating in credit markets are important source of labor market variables fluctuations because linkages between the real economy and credit markets are worth to focus on. Credit shocks are frictions that prevent firms from funding all desired recruitment or investment. This incapacity is assumed to be due to credit constraints or inability to borrow. Credit shock does not mean financial distress or default risk or economic distress, although these things are for sure correlated with credit shocks.

A negative credit shock is identified by assuming that it leads to an increase in the price of credit, here represented by the GZ spread.

4.2 Estimation results and impulse response functions

The structural impulse response functions measure the impact on the level of each $X_t$ variables at time $t + s$ for different values of $s$ of a one-standard deviation shock in the structural errors terms. We assume that this error returns to zero in future periods, and that all other errors are equal to zero. Errors are indeed assumed to be uncorrelated across equations. We use the Cholesky orthogonalised impulse responses to unexpected shocks. It enables to see what are the effects of identified structural shocks on labor and credit markets variables. As there are nine variables in $z_t$, the monetary policy shock, $\epsilon_t^R$, is the tenth element of $\epsilon_t$. And the credit shock is the eleven element of $\epsilon_t$. We analyze a positive shock on
\( \epsilon_t^R \), that corresponds to a contractionary monetary policy shock; as well as a positive shock on the GZ spread, that corresponds to a negative credit shock, meaning that credit is more expensive.

In the following figures, the solid lines correspond to the point estimates and the dashed lines are two standard error confidence band. The horizontal axes indicate quarters. And the examination of the one-standard deviation shocks are made at the horizons up to 24 quarters.

Then, the importance of the effects of structural shocks for the volatility of variables at business cycles frequencies are also studied (variance decomposition of forecast errors). Variance decomposition measures the relative contributions of shocks to the dynamics of endogenous variables.

4.2.1 Impulse responses to a monetary policy shock

The impulse responses to a monetary policy shock are obtained, by using the Baa-Aaa spread and the Normal-Wishard type priors.

The figure 2 illustrates the Cholesky orthogonalised impulse responses to a contractionary monetary policy shock. We examine the impulse response function of all variables in \( X_t \) to a contractionary monetary policy shock. The contractionary monetary policy shock implies as usual an increase in the Federal funds rate until four quarters. It leads to a large hump-shaped decrease of the real GDP. The peak fall in output is about 2% at about two years. Firms need less workers, as the production decreases. The unemployment rate increases as a consequence. It reaches a peak increase of around 3.5% almost eight quarters after the rise in interest rate, through a strong increase in the job destruction rate and a slower decrease in the job creation rate, consistent with the results of Trigari (2009). Job destruction increases up to 1.7% at six quarters and job creation reaches a peak response at 0.8% at about eight quarters. The effect is persistent for both variables. Hours worked declines with a peak about 2.1% at two years, but the decline is less important compared to the increase of unemployment. This evidence is consistent with Ravn and Simonelli (2007) and Hansen (1985), who particularly shows that 55% of the variation in hours worked are due to variations in the number of employed people and only 20% is due to variations in average hours worked. This evidence is also consistent with Trigari (2009) finding that both extensive and intensive margins react after a monetary policy shock. Employment and hours worked per employee fall. Real wages decline slowly with but persistently at 0.5%. They are less reactive than hours worked per employee. The maximum decrease in real wages is about 0.5%. A hump-shaped rise response of inflation is also observed, with a peak effect occurring after about five quarters. The Baa-Aaa spread and the default rate follow the same pattern of evolution. They both hump-shape, with a peak effect occurring after about five quarters. The
Baa-Aaa spread reaches a peak increase of around 0.8% and the default rate of around 1.9%.
At longer forecast horizons, real GDP, unemployment, job creation, job destruction, inflation and Baa-Aaa
spread return to their initial level.
To conclude, the contractionary monetary policy shock causes a persistent economic downturn. Output,
real wages, hours per worker decline and unemployment, credit spreads and default rate increase.

![Figure 5](image)

**Figure 5**: The median response to a one standard deviation positive shock to Federal funds rate. The solid lines correspond to the point estimates. The dashed lines are the 90% upper and lower bands of the credibility interval for each variable.

### 4.2.2 Impulse responses to a credit shock

The impulse responses to a credit shock are obtained by using the GZ spread and the Minnesota type
priors, as in Smets and Wouters (2003) and Smets and Wouters (2007). Moreover, to make our analysis consistent, we have to delete the default rate from our data set. Indeed, as explained by Gilchrist and Zakrajšek (2012), the GZ spread can be decomposed in different parts, including an expected default part. If we keep the default rate, there is a problem of endogeneity.
The credit shock corresponds to an increase in the GZ spread. A hump-shaped decrease of the real GDP
is observed, as well as for inflation and for the Federal funds rate, consistent with the results of Popescu and Smets (2010). The peak fall in output is about 0.8% at 5 quarters. Inflation reaches a lower peak at about 0.15% at 5 quarters too but the impact is more persistent, until three years. The same kind of observation is made for the Federal funds rate, with a peak effect of 0.5% at about seven quarters, and a persistence effect of about seven years. The unemployment rate increases until six quarters. It reaches a peak at 0.2%, through a decrease of job creation and a stronger increase in job destruction. Job creation reaches a peak decline at 0.2% at about four quarters and job destruction increases up to 0.4% at four quarters too. As found by Popescu and Smets (2010), the impact on real GDP is more important than the impact on unemployment. Hours per worker decrease also until twelve quarters, with a peak decrease of 0.8% at 6 quarters. The real wage increases after the increase of credit spreads, reflecting the real cost of labor, including the credit costs to employ this working force.

**Figure 6:** The median response to a one standard deviation positive shock to GZ spread. The solid lines correspond to the point estimates. The dashed lines are the 90% upper and lower bands of the credibility interval for each variable.
5 Conclusion

By integrating frictions in labor and credit markets in a new-Keynesian model, we find that the procyclicality of the risk premium impacts labor market variables, namely, vacancy posting decisions, wages and unemployment. Asymmetric information on credit markets is the source of lower posting vacancies and a higher unemployment level. A financial mark-up appears in the model, charged by financial intermediaries in order to overcome the asymmetric information in credit markets. This mark-up is transmitted through wholesale firms, and their prices, to the labor market. Thus, the existence of frictions in credit markets generate through a pricing channel all the more frictions in labor markets. Our structural BVAR analysis illustrates some of our theoretical results. Notably, after a credit shock on the risk premium, we observe a hump-shaped increase of unemployment. Furthermore, an increase of real wages appears, represented the higher marginal costs incurred by wholesale-firms due to the increase of the financial mark-up.
References


