Strategic Choices in Polygamous Households: Theory and Evidence from Senegal

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[Preliminary and incomplete. Please do not quote]

Abstract

This paper proposes a new framework to account for fertility choices in polygamous households. I specify the main drivers of fertility in the African context and I model explicitly how the fertility of one wife impacts the behavior of her co-wives. Each wife cares about her relative number of children, compared to the others, while the husband cares about his total number of children with all wives. Moreover, marriage duration matters, because couples have only some imperfect control over births. I solve for the optimal fertility of wives in bigamous households using the concept of Cournot-Nash equilibrium. The model implies that the timing of successive marriages will impact completed fertility as well as birth spacing patterns. Exploiting this feature, I derive predictions to test empirically for the existence of strategic interactions. Combining original data from a household survey and the Demographic and Health Surveys in Senegal, I show that children of both wives are strategic complements : one wife will raise her fertility in response to an increase by the other wife. The reproductive rivalry between co-wives has been widely documented in the anthropology and sociology literature, but it had never been established empirically so far. By providing new insights into potential obstacles to the fertility transition, this paper has strong implications for population policies in Africa. It also contributes to the literature on household behavior as one of the few attempts to open the black box of non-nuclear households.

Keywords : Noncooperative models, Polygamy, Fertility, Africa.

JEL Codes : C72, D13, J13, J16, O15, O55.

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Until recently, economists have considered households as unitary entities. Individual resources are assumed to be pooled, and individual preferences are somehow aggregated to form a single utility function. The unitary model has achieved great success because it generates simple comparative statics and predictions on the impact of economic policy on household behavior and welfare. However, a number of researchers have argued that the unitary model does not stand up well to empirical testing and lacks robust theoretical foundations (Alderman, Chiappori, Haddad, Hoddinott, and Kanbur 1995). In recent years, a growing literature has developed alternative approaches recognizing that households consist of different members having their own preferences. These non-unitary models then differ on how decisions are actually taken within the household. On the one hand, collective models assume that, whatever the decision process, outcomes are Pareto efficient. On the other hand, strategic (or non-cooperative) models use the concept of Cournot-Nash equilibrium – each member maximizes her own utility taking the actions of others as given – and typically predict an inefficient outcome (see Chiappori and Donni (2009) for a survey of the literature).

In African countries, where household structure is generally not nuclear, the non-unitary approach is all the more relevant. Strategic models appear to be suitable for complex household structures, whereas collective models may be ruled out by various sources of inefficiencies such as social norms and information asymmetry between the spouses. Indeed, the few studies testing if household members achieve efficient outcomes in the African context have concluded that they do not (Udry (1996) on agriculture production in Burkina Faso, Dercon and Krishnan (2000) on risk sharing in Ethiopia, Duflo and Udry (2004) on resource allocation in Cote d'Ivoire, Ashraf, Field, and Lee (2014) on fertility choices in Zambia).

This paper shows that a simple strategic model describes well fertility choices in polygamous households. [1] I focus on polygamous households because little effort has been devoted

1. Polygamy is a marriage that includes more than two partners. It encompasses both polygyny, in which a man has several wives, and polyandry, in which a woman has several husbands. Throughout this paper, I use the term polygamy to refer to the former situation, which is by far the most practiced form of polygamy.

so far to understanding the decision process in this specific setting, although interactions between members are likely to be particularly rich. Polygamous households represent a non-negligible share of the African population. More than 10% of married men have several wives in 28 countries in Sub-Saharan Africa ; this proportion raises up to 40-50% in Burkina Faso, Cameroon and Senegal (Tertilt 2005). The term "polygamy belt" has been coined to define an area between Senegal in the west and Tanzania in the east, in which it is common to find that more than one third of married women are engaged in a polygamous union (Jacoby 1995).

I chose fertility as the outcome of interest because, for many parents in developing countries and especially in Africa, offspring are their economic futures. Children mitigate the lack of insurance markets and social safety nets by taking care of parents in old age and during times of need. Banerjee and Duflo (2011) state that, for the poor, children represent "an insurance policy, a savings product as well as some lottery tickets". In particular, they are at the core of African women's lives. On the one hand, women's status, and their security in case of widowhood or abandonment by the husband, critically depend on their ability to have children (Bledsoe 1990). On the other hand, women put their live in jeopardy each time they give birth : in Sub-Saharan Africa, the lifetime risk of maternal death is 1 in 38 (WHO, UNICEF, UNFPA and The World Bank 2014), and maternal mortality is one of the two main causes of female excess mortality (Anderson and Ray 2010). Setting the right pace for births is probably one of the major challenge faced by African women. Moreover, children are the main stake in polygamous unions. The reproductive rivalry between co-wives has been widely documented in the anthropology and sociology literature (Jankowiak, Sudakov, and Wilreker 2005), but it has not been shown empirically. What has been established by demographers is the negative correlation between fertility and polygamy : women engaged in polygamous unions tend to have fewer children than other women (Lesthaeghe 1989). This empirical result is at first sight in contradiction with the idea of co-wives overbidding for children.

In this paper, I set up a simple model in which the wife chooses the optimal fertility taking into account her own preferences, the preferences of her husband and the duration of the couple's reproductive period. Demographers have shown that this duration, which is determined by husband's and wife's ages at marriage, influences significantly completed fertility in the African context, where couples have only some imperfect control over births. I further build on the demography literature to assume that in the absence of conscious manipulation, births would occur at a lower rate in polygamous unions than in monogamous ones. Next, the model describes how the fertility of one co-wife impacts the choices of the other wife in bigamous unions. On the one hand, it is negatively correlated with the number of children targeted by the husband with the index wife, because he cares about his total number of children. On the other hand, it correlates positively with the wife's objective, because she cares about her relative number of children. Eventually, I find a closed form for the number of children of both wives at the Nash equilibrium. For the first wife, I also express the difference in equilibrium birth spacing before and after the arrival of the second wife, taking into account her anticipations.

In the model, children of both wives are strategic complements if and only if the "co-wife rivalry" effect dominates the "husband" effect. In this case, an increase in the equilibrium fertility of one wife raises the fertility of the other. Considering the timing of successive marriages as predetermined, I am able to derive predictions to test empirically for the existence of strategic complements. First, the completed fertility of one wife should increase with the duration of the other wife's reproductive period. Second, birth spacing should be the same for monogamous and first wives as long as the second wife has not arrived. Third, on average, first wives should lengthen birth spacing after the arrival of the second wife, but they should lengthen less when the second wife's reproductive period is long. Fourth, second wives should have shorter birth spacing when first wives have spent more time as the single wife, and have more time left before the end of their reproductive period.

Combining original data from a household survey and the Demographic and Health Surveys in Senegal, I confirm each prediction. I carry out the tests on Senegalese data, but my model may fit with the reality of other countries as soon as co-wives have (i) competing reproductive interests, and (ii) some imperfect control over fertility. Both conditions seem likely to hold throughout Africa, but further research is needed to determine to what extent the behavior of polygamous households is culturally specific.

The outline of the paper is as follows. Section 1 provides background on fertility in polygamous unions. Section 2 presents the Senegalese data and some descriptive statistics. Section 3 sets up the model and derives testable predictions. Section 4 reports the results of the empirical tests, Section 5 discusses the implications for public policies, and Section 6 concludes.

1 Polygamy and Fertility

1.1 Qualitative evidence : co-wife reproductive rivalry

Conflicts between co-wives are pervasive in polygamous societies : co-wife rivalry is a recurring theme in African novels[2] and it has been thoroughly studied by anthropologists and sociologists working on polygamous ethnic groups.

Jankowiak, Sudakov, and Wilreker (2005) gathered information on co-wife interactions in 69 polygamous systems from all over the world (among which 39 are in Africa) to identify the determinants of co-wife conflict and cooperation. They conclude that conflict is widespread and primarily caused by competing reproductive interests. They note that ”reproductive vitality, women's age in the marriage, and the presence or absence of children influence a woman's willingness to enter in or avoid forming some kind of pragmatic cooperative

2. See for instance books by China Achebe, Sefi Atta, Mariama Ba, Fatou Diome, Buchi Emecheta, Aminata Saw Fall and Ousmane Sembene.

relationship with another co-wife." Thus, conflict is less prevalent when one wife cannot have children.

In the African context, Fainzang and Journet (2000) have documented that wives in polygamous unions overbid for children. Women commonly resort to marabouts to increase their own chances to get pregnant and to cause infertility or stillbirths for the co-wife. In most extreme cases, aggressions may jeopardize children's lives. Indeed, child mortality is found to be higher in polygamous households and co-wife rivalry is considered as one important risk factor (Strassman 1997, Areny 2002).

There are many reasons why wives care so much about their number of children, relative to the number of children of their co-wives. This difference defines social status, authority over co-wives and husband's respect (Fainzang and Journet 2000). It may also be interpreted as a sign of husband's sexual and emotional attention, which clearly matters for the wife's well-being when jealousy is rife (Jankowiak, Sudakov, and Wilreker 2005). In an economic perspective, the relative number of children directly determines the wife's share in husband's resources. It is particularly important in case of widowhood because in most African countries, a man's bequest is to be shared among his children. The surviving wives generally have little control over inheritance other than through their own children (United Nations 2001, Lambert and Rossi 2014). In polygamous unions, each wife needs therefore to ensure that enough children are born to secure current and future access to husband's resources.

1.2 Quantitative evidence : lower fertility in polygamous unions

Although the competition between co-wives for more children has been qualitatively documented, quantitative evidence is very scarce. Demographers have been working on the relationship between polygamy and fertility for a long time, but their theoretical framework does not take into account strategic behaviors. They assume that a regime of natural fertility prevails in Sub-Saharan Africa. It implies that a woman's fertility is mostly determined by biological constraints and social norms influencing the length and the intensity of her exposure to the risk of pregnancy : men's and women's ages at marriage, imperatives for widows or divorcees to remarry, breastfeeding practices, post-partum taboos etc. This framework leaves little room for individual choices.

Demographers working on Sub-Saharan Africa have established that, at the woman level, fertility is lower in polygamous unions than in monogamous ones (see Chapter 7 by Pebley and Mbugua in Lesthaeghe (1989) or Garenne and van de Walle (1989) and Lardoux and van de Walle (2003) for specific studies on Senegal). This empirical regularity is first explained by infertile unions. On the one hand, women with low fecundity are over-represented in polygamous unions because husbands cannot be satisfied with an infertile wife. On the other hand, polygamous marriage may be a way for widowed or divorced women to fulfill the obligation to remarry. In such "safety nets" unions, spouses generally do not aim at giving birth to children. This composition effect partly explains the difference in fertility between monogamous and polygamous unions. The second explanation is the difference in the timing of marriage : junior wives tend to get married older, and to an older husband. Consequently, these couples have shorter reproductive periods, which mechanically translates into fewer children in a natural fertility regime. The last explanation is about the frequency of intercourse, which is lower in polygamous unions. Indeed, a rule of rotation between the wives for conjugal obligations is implemented and highly monitored. Wives have to share the husband's bed time, which lengthens the period of time before getting pregnant. This is especially true when spouses do not co-reside and the husband pays his wives regular visits. Also, the non-susceptible period following the birth of a child is longer in polygamous unions because the availability of alternative partners makes it easier for husbands to observe the post-partum abstinence.

Interestingly, some results in the study by Lardoux and van de Walle (2003) on Senegalese data cannot be rationalized in the natural fertility framework. First, the authors expected that post-partum abstinence would reduce the likelihood of simultaneous births by co-wives and foster alternate births. In fact, they find a strong positive association between each wife's probability of child-bearing during a given year. Strategic choices may well explain this pattern : if the wife targets a relative number of children, she gets pregnant as soon as her rival does to keep pace with her. The other unexpected result is that the presence of a wife who is past her fecund years impacts positively the fertility of the younger wives. The authors had hypothesized the opposite relationship assuming that the older wife would claim her share of bed time and enforce the compliance with intercourse taboos. In a strategic framework, this might be interpreted as a hint that junior wives are catching up with an older wife who already gave birth to many children.

To my knowledge, there is no empirical study providing evidence that fertility choices of a given wife impacts the choices of her co-wife. This paper attempts to fill in this gap.

2 Data and Descriptive Statistics

2.1 Data

Empirical tests are carried out on two Senegalese surveys that are strongly complementary.

2.1.1 Poverty and Family Structure (PSF)

On the one hand, I exploit original data from a household survey entitled "Poverty and Family Structure" (PSF) conducted in Senegal in 2006-2007. [3] It is a nationally representa-

3. Detailed description in Vreyer, Lambert, Safir, and Sylla (2008). Momar Sylla and Matar Gueye of the Agence Nationale de la Statistique et de la Démographie of Senegal (ANSD), and Philippe De Vreyer (University of Paris-Dauphine and IRD-DIAL), Sylvie Lambert (Paris School of Economics-INRA) and Abla Safir (now with the World Bank) designed the survey. The data collection was conducted by the ANSD.

tive survey conducted on 1,800 households spread over 150 primary sampling units drawn randomly among the census districts. About 1,750 records can be exploited.

The main advantage is that all spouses of the household head were surveyed, even if they do not co-reside. This is very important because approximately one fourth of women do not live with their husband. Standard household surveys only gather information on co-residing spouses, so information is missing in incomplete unions, and the sample of complete unions is selected. Here, I have information on husband and *all* co-wives, whatever the residence status. There remain two selection biases : the sample is restricted (i) to household heads and their spouses, and (ii) to currently married people. [4]

The goal of the survey was to obtain, in addition to the usual information on individual characteristics, a comprehensive description of the family structure. In particular it registered the dates of birth for all living children below 25, even if they do not live with their parents. Children who died are also reported but there is no information on the timing of deaths. As a result, a woman's complete birth history for surviving children is available only if all her children are under 25 years old.

Moreover, detailed information is collected on the marital history of all spouses : age at first marriage, date of current union, having or not broken unions, date of termination of latest union. Therefore, I am able to retrace the timing of marriages and to identify children from previous unions. Lastly, I use information on education, occupation and income, as additional controls.

The PSF sample consists of 1,317 unions : 906 monogamous unions and 411 polygamous unions, among which 321 with two wives, 66 with three wives and 24 with four wives. Roughly one half of women are engaged in a polygamous union, a proportion similar to the one computed on DHS below and in line with demographers' estimations (see Chapter 27 by Antoine in (Caselli, Vallin, and Wunsch 2002)).

4. There is very limited information on co-widows.

2.1.2 Demographic and Health Surveys (DHS)

On the other hand, I exploit DHS surveys (Demographic and Health Surveys) collected in Senegal in 1992, 1997, 2005 and 2010. [5] DHS data contain stratified samples of households in which all mothers aged 15 to 49 are asked about their reproductive history. In particular we know the date of birth and death of each of their born children, along with mothers and children's characteristics. Male questionnaires are further applied to all eligible men in a subsample of these households. [6] As a result, it is possible to match mother's recode and male's recode in order to create a couple's record. DHS data are provided with survey weights to ensure that the survey sample is representative of all co-residing couples in a given age group at the country level. Nonetheless, the sample is not representative of all unions because a spouse is not surveyed if (i) she does not co-reside, (ii) she is above the age limit (50 for women, 60 for men in the last two waves). As a result, out of 30,655 women in the mother's recode, only 6,174 have a match with a male questionnaire.

The DHS sample is much more selected than the PSF one, but DHS collect relevant data that is missing in PSF. In particular, they contain information on fertility preferences of men and women. Respondents are asked how many children they would like to have, or would have liked to have, in their whole life, irrespective of the number they already have. They are also asked about their ideal gender composition. Moreover, we know a woman's complete birth history, including children who left the household or who are dead at the time of the survey.

The DHS sample consists of 5,254 unions : 3,614 monogamous unions and 1,640 polygamous unions, among which 1,263 with two wives, 316 with three wives and 61 with four

5. Three other surveys were conducted in Senegal in 1986, 1999 and 2012-13, but I do not consider them, because in 1986 and 2012-13, there is no information on men, and in 1999, the quality of data does not meet the criteria of standard DHS.

6. In 1992 and 1997, men should be older than 20 to be eligible, whereas in 2005 and 2010, they should be between 15 and 59 years old. The proportion of households selected to administer male questionnaires was 33%, except in 1997, when it was 75%.

wives. If I focus on bigamous unions, both wives are found in the sample in 46% of the cases (579 unions). Even in this case, information on the timing of unions is often missing. I only know the date of first marriage, so I am able to deduce the timing of successive marriages only when both wives are in their first marriage (246 unions).

2.2 Descriptive statistics on fertility in current union

I compute the descriptive statistics on completed fertility using the PSF data. I restrict the sample to women over 45 years old, who have reached the end of their reproductive life, in order to avoid censoring issues. I consider only children born in the current union.

In line with demographers' concept of natural fertility, I find that the length of couple's reproductive period is an important driver of women's total fertility. In Table 1, I test whether husband's and wife's age at marriage are negatively correlated to the number of children, controlling for the union status, and differentiating between spouses with a small and a large age difference.[7] The idea of the test is that, if the age difference between spouses is small, the length of the reproductive period should be driven by the wife's age at marriage, not by the husband's age at marriage. Indeed, in this case, the wife is expected to reach the end of her reproductive life sooner than the husband. If the age difference between spouses is large, it should be the opposite. Signs are in line with predictions : when the age difference is small (resp. large), only the wife's (resp. husband's) age at marriage is significantly, negatively correlated to the number of children. This is the reason why I consider the length of couple's reproductive period proxied by T = min (45- wife's age at marriage ; 60-husband's age at marriage) in empirical tests.

Figure 1 shows the distribution of the final number of children for monogamous, senior and junior wives. Junior wives stand out with a large proportion (30%) having no child with

7. I used thresholds of 15 and 18 years, which correspond roughly to the difference in the ages of fertility decline between men and women.

their current husband. Those women are probably engaged in a kind of safety net union. The proportion of childless women is the same for monogamous and first wives. It contradicts the hypothesis that husbands would take another wife when the first one is infertile ; it rather supports the idea that husbands would repudiate an infertile wife, who would then ends as a junior wife in another union. Monogamous wives tend to have slightly more children than first wives, but the left and right tails of the distribution are similar in the two groups.

Table 2 confirms and specifies these remarks. Women engaged in a polygamous union have, on average, 1 child fewer than in a monogamous union. The whole gap is driven by junior wives : they have, on average, 2.5 children fewer. As for senior wives, they have the same number as monogamous ones. Roughly half of the gap between junior wives and the others is explained by infertile unions. When I restrict the sample to women having at least one child with their current husband, the difference decreases down to 1.4 children. Another explanation put forward in the literature review is the difference in ages at marriage. After controlling for the length of couple's reproductive period, there remains a gap of approximately 1 child. This last figure remains unchanged if I control for having children from previous unions.

If I disentangle the results by mother's rank in polygamous unions, I find that fertility decreases as the number of wives increases. The first column of Table 3 shows that, controlling for T and children from previous unions, and excluding childless women, the number of children decreases with the mother's rank. Findings are similar when I consider birth spacing. In column 2, I report the estimates of a duration model similar to the one specified in the empirical section, and I find that the higher the rank of the mother, the longer the durations between births. [8] And senior wives have longer birth spacing than monogamous wives.

Turning to men's total fertility, it is more complicated to avoid censoring issues because

8. Nonetheless, in both regressions, the difference is only significant between first wives and junior wives, not among junior wives. There are few women of rank 3 and 4 so the estimates are very imprecise.

older men may still have the opportunity to take another fertile wife. When I restrict the sample to unions in which all wives are above 45, and the husband is above 60, figures are consistent with statistics computed on women's side. On average, monogamous men have 5 children, men with two wives 7.6 children, and men with three wives 9.7 children.[9]

2.3 Descriptive statistics on preferences

I compute the descriptive statistics on fertility preferences using the most recent DHS waves (2005 and 2010), which were conducted in the same years as the PSF survey (2006-2007). As shown in Table 4, preferences of men and women differ considerably. Women would like to have, on average, 5.7 children, whereas men want 9 children. Medians are respectively 5 and 7. Within couples, a husband want on average 3.1 children more than his wife. Preferences are aligned in only 13% of the cases ; in almost two thirds of couples, the husband wants strictly more children than his wife.

A sizeable proportion of the respondents gave a non-numerical answer to the question "How many children would you like to have, or would you have liked to have ?". 23% of women and 31% of men answered "I don't know" or any non-numerical statement such as "It is up to God". As a result, I know the ideal number of children for both spouses in a bit more than one half of the couples (55%). The selection is not random : those couples are more "westernized"[10] than couples in which at least one spouse has a non-numerical ideal family size.

One may wonder whether respondents rationalize ex-post their fertility behavior, and report that they would have wanted exactly the same number of children as they actually

9. I do not show the average for unions with four wives because there are too few data points.

10. Table A.1 in Appendix shows that couples in which both spouses report her ideal number of children are more urban, richer and more educated. The proportion of Christians is larger, as well as the proportion of monogamous unions. They belong to younger cohorts and got married older. There is also some variation across regions - they are more likely to live in Dakar, Saint-Louis, Fatick, Kolda or Kaffrine, and less likely to live in Diourbel, Kaolack, Thies or Louga - and across ethnic groups - the proportion of Serer and Jola is larger while that of Fula is smaller.

have had. This would be an issue, because the reported ideal number would be driven by the outcome of the whole decision process, and would therefore not be a reliable measure of innate preferences. To assess the validity of such a concern, I restrict the sample to older couples (all wives above 40, husbands above 50) and I compare the ideal family size to the realized one. Ideal and realized numbers coincide for only 10% of husbands and 17% of wives. Over one third of men and one half of women declare that they would have wanted fewer children than they actually have had. Such figures provide some level of assurance that the scope for ex-post rationalization is limited.

Table A.2 in Appendix reports the predictors of the ideal number of children, for men and women separately. First, the level of socio-economic development is clearly negatively correlated with the ideal family size : educated and wealthier men and women, as well as urban men, want fewer children. This is also the case for younger cohorts. Then, marital history matters : men and women who got married younger, and men who got married more than once, want more children. Household heads and their wives display on average the same preferences as other members.[11] Last, there is some variation across religions – the ideal family size is smaller for Christians – ethnic groups and regions. Since there is no information on preferences in PSF, I will control for this wide set of predictors in order to reduce as well as possible the omitted variable bias.

Another interesting result from Table A.2 is that there is no difference between the preferences of wives engaged in a monogamous or a polygamous union. I further perform additional tests to support the idea that a woman's marital status is orthogonal to her ideal number of children. In polygamous unions, there is no difference between senior and junior wives. And the status (monogamous, senior, junior) does not predict the likelihood to report

11. Since the PSF sample is restricted to household heads, I ran the same regression on the sub-sample of household heads in DHS. Predictors of the ideal number children are the same as in the whole sample, except that age at first marriage and being in first marriage are no longer significant for men, while being employed becomes significant for women.

a numerical ideal family size. [12] In contrast, the ideal number of children is much larger for polygamous husbands than for monogamous ones. If I consider the raw average, monogamous husbands want 7.6 children against 12.2 for polygamous husbands. The gap goes down to 2.7 children when I control for the whole set of predictors, but it remains highly significant. So there is a strong, positive correlation between a man's taste for a large family and his likelihood to take a second wife. [13]

2.4 Descriptive statistics on the timing of unions

The timing of unions plays a key role in my model of fertility choices. Table 5 provides some descriptive statistics to keep in mind. Women tend to marry much older husbands : the median age at marriage is 17 for first wives against 28 for their husbands. The second marriage generally takes place around 12 years later : the husband is 40, the first wife 29, and the second wife 22. But the variation in the timing of the second marriage is large : in the first quartile, it takes place within the first 6 years of marriage, whereas in the last quartile, it takes places after 16 years of marriage. First wives may be still very young when their rival arrives (below 24 years old in the first quartile) or already quite old (above 36 in the last quartile). The situation of second wives is even more diverse. Around one third have already been married, which explains why the age at marriage is so large in the last quartile (31 years old), while others are very young (below 17 in the first quartile).

Among junior wives, it is important to distinguish between those who may overbid for children and those who will not compete. Some characteristics may be used to identify unions that are first and foremost a safety net. For instance, if the junior wife was older than 45 years old as she arrived, or if she is much older than the first wife and has already been

12. Then, within polygamous unions, the correlation between preferences of co-wives is positive, but not very high (0.28). Preferences coincide in 19% of the cases. I compute the statistics on bigamous unions, in which both wives belong to the sample and report an ideal number of children (204 unions).

13. Interestingly, the gap between the ideal family size and the realized one is the same for monogamous and polygamous husbands.

married before, she is probably not a competitor. In my sample, such characteristics are found in 12% of the cases (51 unions). In the vast majority of these unions, not a single child was born.

In the theoretical part, I consider a model with two players ; it may correspond to unions with two, three or four wives, provided that exactly two wives are actually competing for children. As a consequence, I will generally focus on unions with exactly two competitors in the empirical tests. They account for around three quarters of polygamous unions.

3 The Model

3.1 A simple model of fertility choices

The model of fertility decisions is in continuous time, the decision-maker is the woman, and the choice variable is the birth rate, λ. At date T, the couple reaches the end of the reproductive period with $n = \lambda.T$ children.[14] In concrete terms, it means that women choose to give birth every x years, where $x = \frac{1}{\lambda}$.

There is no uncertainty in this framework : λ is a frequency and $\lambda.T$ is the realized number of children. I do not explicitly model the intertemporal evolution of fertility choices and outcomes, although a dynamic stochastic model would better reflect the true decision process. There is a trade-off between realism and tractability. Most dynamic stochastic models do not generate closed-form solutions and the predictions derived from comparative dynamics vary from model to model (see the survey by Arroyo and Zhang (1997)). Since the aim of this paper is to evidence noncooperative behaviors, I chose the simplest framework that allowed me to model and test for the existence of strategic interactions.

Fertility choices are determined by three drivers. First, women face a standard economic

14. The number of children is therefore not necessarily an integer. Leung (1991) proposed to consider the number of children as a flow of child services in efficiency units when a continuous measure of family size is needed.

trade-off modeled by a function $v(n, n_w^{id})$, that captures the net gain of having n children for the mother ; I assume that $v(.)$ has an inverted U-shape and reaches its maximum when $n = n_w^{id}$, the mother's ideal number of children. Second, even if the wife is the only decision-maker, husband's preferences over fertility influence the outcome by entering the wife's utility. The idea is that husbands may punish their wives if the average birth spacing is far from their expectations, and that wives are aware of such a threat. So women incur a cost to deviate from the husband's ideal number of children, n_h^{id}. I denote the cost function $H(n, n_h^{id})$ and I further assume that it has a U-shape and reaches its minimum when $n = n_h^{id}$. As shown in descriptive statistics and in line with the literature on Africa (Ashraf, Field, and Lee 2014), n_h^{id} is generally greater than n_w^{id}. Third, I build on demographers' concept of natural fertility to introduce a natural birth rate, λ^{nat}. It reflects the fact that, women's choices are constrained by biological and social norms. I further define a function $N(T.(\lambda - \lambda_m^{nat}))$ capturing the cost to deviate from the natural level. It has an U-shape in λ and reaches its minimum when $\lambda = \lambda^{nat}$. The argument in the cost function is $\int_0^T (\lambda - \lambda^{nat}) dt$; it corresponds to the accrual of instantaneous deviations during the whole reproductive period. One can think of n_i^{id} as the ideal number of children that individual i would like to have if the conception of children was free from any biological constraint.

3.1.1 In monogamous societies

In monogamous societies, a wife chooses λ maximizing $u(n_w^{id}, n_h^{id}, \lambda_m^{nat}, \lambda.T)$. I further assume that the wife's utility is separable in three components :

$$u(n_w^{id}, n_h^{id}, \lambda_m^{nat}, \lambda.T) = v(\lambda.T, n_w^{id}) - \theta^h H(\lambda.T, n_h^{id}) - \theta^n N(T.(\lambda - \lambda_m^{nat})) \tag{1}$$

$\theta^h \geq 0$ and $\theta^n \geq 0$ capture the intensity of marital and natural constraints, respectively. Payoffs are paid at the end of the reproductive period. This is important to ensure time consistency.[15] I further note $n^{nat} = T.\lambda_m^{nat}$.

A natural way to put more structure on this optimization problem is to assume that the wife minimizes a weighted sum of distances : distance to her ideal number (weight 1), distance to the husband's ideal number (weight θ_h), and distance to the natural number (weight θ_n). That is why I consider the following parametric forms :

$$v(n, n_w^{id}) = -(n - n_w^{id})^2 \text{ and } H(n, n_h^{id}) = (n - n_h^{id})^2 \text{ and } N(n, n^{nat}) = (n - n^{nat})^2$$

The first order condition gives an optimal birth rate :

$$\lambda^* = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n n^{nat}}{(1 + \theta^h + \theta^n).T} \tag{2}$$

Hence the optimal number of children, $n^* = \lambda^*.T$, is a weighted average of n_w^{id}, n_h^{id} and n^{nat}. The optimal birth rate and the optimal number of children increase with n_w^{id}, n_h^{id} and λ_m^{nat}. An increase in T raises the final number of children, but reduces the birth rate. Last, the impact of a variation in θ^h and θ^n depends on the relative size of n_w^{id}, n_h^{id} and n^{nat}.

From this very simple model, I derive a testable implication.

Prediction 1 *In monogamous societies, the final number of children is a weighted average of the preferences of the wife, the preferences of the husband and a natural number proportional to marriage duration.*

The first column of Table 6 shows that my framework is relevant. The three drivers are

15. At date 0, the woman solves the maximization problem and chooses λ^*. Suppose that she can update her choice at date t. She already has $\lambda^*.t$ children, and maximizes over λ' :

$$v(\lambda^*.t + \lambda'.(T-t), n_w^{id}) - \theta^h H(\lambda^*.t + \lambda'.(T-t), n_h^{id}) - \theta^n N(t.(\lambda^* - \lambda_m^{nat}) + (T-t).(\lambda' - \lambda_m^{nat}))$$

It is equivalent to maximizing over $\mu = \frac{\lambda^*.t+\lambda'.(T-t)}{T}$:

$$v(\mu.T, n_w^{id}) - \theta^h H(\mu.T, n_h^{id}) - \theta^n N(T.(\mu - \lambda_m^{nat}))$$

Therefore $\mu = \lambda^* = \lambda'$. The woman is time-consistent.

significantly correlated to total fertility, accounting for 36% of the variance. The constant is not significantly different from zero. [16] Regressing the number of children on n_w^{id}, n_h^{id} and T allows me to estimate the structural parameters of the model. The estimation I get for the natural birth rate is sensible : 1 birth every 3 years during the whole reproductive period. Given that $T = 26$ years on average, it corresponds to approximately 8 children for the average couple. [17] This natural number of children significantly constrains fertility choices : θ^n is estimated to be around three. Last, the order of magnitude of θ^h is one half, but I cannot reject the hypothesis that the preferences of the husband and those of the wife have the same weight.

When I turn to polygamous unions, in column two, the fit is not as good. The R^2 drops to 0.14 and the preferences of the wife are no longer significant, so that I can no longer get consistent estimates of the parameters. Something is missing to account for a woman's choices in a polygamous union, and I claim in the next section that the missing element is the fertility of her rival. As suggestive evidence, I added the preferences of the rival as an additional driver. [18] The correlation is large and positive, although not significant ; more importantly, the R^2 increases up to 0.34, which is close to the level observed in monogamous unions.

3.1.2 In bigamous societies

In bigamous societies, the timing is split into two periods. In the first period, a monogamous couple is formed. The wife takes into account her expectations about the arrival of a

16. Yet, the constant is rather large in magnitude and negative. One explanation is that I consider women between 40 and 50 years old who may still have an additional child. There are not enough observations to restrict the sample to an older age bracket.

17. This number is consistent with estimates produced by the founding father of the natural fertility concept, Louis Henry. Using data from various populations in the world, he concludes that the completed fertility for a woman married at 20 years old is between 6 and 11. It varies from one population to another, and seems to be higher in Europe than in Africa and Asia (Henry 1961).

18. I observe the preferences of the rival only in the case of bigamous unions in which both wives are found, which leaves me with a very small sample (60 observations).

co-wife when she chooses λ_0. The second period starts at $t = S$ when the second marriage takes place. If $S = +\infty$, the wife always remains in a monogamous union. Otherwise, the two wives play a simultaneous, non-cooperative game. Payoffs are paid when both reproductive periods are over.

One important assumption in this setting is that the occurrence and timing of the second marriage are exogenous to fertility choices. One may argue that husbands are likely to take another wife to mitigate first wives' infertility. This is one reason why, as mentioned above, I exclude infertile unions from the theoretical and empirical analysis. Once I focus on unions with two fecund wives, the exclusion restriction holds if men are "all potential polygamists" as stated by Antoine and Nanitelamio (1995).[19] Marrying a second wife seems to be first and foremost a question of opportunity.

In bigamous societies, the utility function of wife i is given by :

$$u(n_w^{id}, n_h^{id}, n_i^{nat}, n_i, n_j) = v(n_i - \epsilon_i n_j, n_w^{id}) - \theta_i^h H(n_i + n_j, n_h^{id}) - \theta_i^n N(n_i - n_i^{nat}) \qquad (3)$$

Where n_j is the final number of children of the rival. It enters directly the utility function of wife i along two dimensions. On the one hand, the husband cares about his *total* number of children $(n_i + n_j)$. For him, the children of the first wife and the children of the second wife are perfect substitutes. On the other hand, the wife cares about her *relative* number of children $(n_i - \epsilon_i n_j)$. As explained in the literature review, the number of children of a wife, relative to the number of children of her co-wife, determine her status as well as her share in husband's resources. The parameter ϵ is meant to capture the intensity of co-wife rivalry.

I also define $n_1^{nat} = \lambda_m^{nat}.S + \lambda_p^{nat}.(T_1 - S)$ and $n_2^{nat} = T_2.\lambda_p^{nat}$. Building on the demography literature, I assume that $\lambda_m^{nat} > \lambda_p^{nat}$. It is important to note that, in the basic model, I

19. Using a biographical approach in Dakar, they show that few men characteristics influence the probability to become polygamous. Approximately one half of surveyed men have been polygamous at some point or another, but no socio-economic factor could predict the event.

consider only the case when $S < T_1$, meaning that first wives have some time left to update their birth rate in response to the arrival of second wives.[20] To sum up, if we compare the first and the second periods, keeping everything else constant, polygamy is associated with (i) a positive shock on wife's ideal number of children, (ii) a negative shock on husband's ideal number of children, and (iii) a negative shock on the natural birth rate.

In the second period, the second wife chooses λ_2 maximizing :

$$u(\lambda_2.T_2, n_1) = v(\lambda_2.T_2 - \epsilon_2 n_1, n_w^{id}) - \theta_2^h H(\lambda_2.T_2 + n_1, n_h^{id}) - \theta_2^n N(T_2.(\lambda_2 - \lambda_p^{nat}))$$

Let me call λ_2^* the optimal birth rate of second wives, and $n_2^* = \lambda_2^*.T_2$, their optimal final number of children.

Turning to the first wife, let me denote λ_0 the birth rate chosen in the first period. At date S, the first wife has $\lambda_0.S$ children, and she is able to update her choice. Her final number of children is given by $n_1 = \lambda_0.S + \lambda_1.(T_1 - S)$. So she maximizes over λ_1 :

$$u(\lambda_0.S + \lambda_1.(T_1 - S), n_2) = v(\lambda_0.S + \lambda_1.(T_1 - S) - \epsilon_1 n_2, n_w^{id}) -$$
$$\theta_1^h H(\lambda_0.S + \lambda_1.(T_1 - S) + n_2, n_h^{id}) - \theta_1^n N(S.(\lambda_0 - \lambda_m^{nat}) + (T_1 - S).(\lambda_1 - \lambda_p^{nat}))$$

It is equivalent to maximizing over $\mu = \frac{\lambda_0.S + \lambda_1.(T_1 - S)}{T_1}$:

$$v(\mu.T_1 - \epsilon_1 n_2, n_w^{id}) - \theta_1^h H(\mu.T_1 + n_2, n_h^{id}) - \theta_1^n N(T_1.(\mu - \frac{\lambda_m^{nat}.S + \lambda_p^{nat}.(T_1 - S)}{T_1}))$$

Let me call $\lambda_1^*(\lambda_0)$ the optimal birth rate of first wives after the shock.

At $t = 0$, the first wife has to choose λ_0 that maximizes her expected utility given her beliefs about the occurrence of the shock, and in case of shock, about the timing of the shock.

20. In an extension of the model, I include the case when $S \geq T_1$ which corresponds the a sequential game (cf. Section 3.4.2).

I assume that the first wife believes that her husband will eventually take another wife with probability π, and that he will remain monogamous with probability $(1-\pi)$. She maximizes over λ_0 :

$$(1-\pi) \times u(\lambda_0.T_1, 0) + \pi \times \mathbb{E}[u(\lambda_0.S + \lambda_1^*(\lambda_0).(T_1 - S), n_2)]$$

The second term in the expected utility is unknown at $t = 0$ because it depends on S and on the second wife's characteristics, in particular her preferences and her age.

3.2 Finding the equilibrium in bigamous unions

I solve the problem by backward induction : focusing on the second period, I determine the best response of each wife, in order to compute the equilibrium of the static game. Then I turn to the first period and determine the optimal initial birth rate.

3.2.1 First stage : best responses

I start by computing the functions of best response taking the number of rivals as exogenous. I replace $v(.), H(.), N(.)$ by the parametric forms and take the first-order condition for second wives. I find :

$$\begin{aligned} n_2^* = \lambda_2^*.T_2 = \frac{n_w^{id} + \theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1 + \theta_2^h + \theta_2^n} &+ n_1.\frac{\epsilon_2 - \theta_2^h}{1 + \theta_2^h + \theta_2^n} \\ &= n_2^{NS} + n_1.B_2 \end{aligned}$$

The term n_2^{NS} corresponds to the optimal choice of second wives if they were not strategic. The term B_2 captures the strategic response to n_1. If $\epsilon_2 > \theta_2^h$, then $B_2 > 0$ and n_2 is increasing in n_1. It implies that children of both wives are strategic complements iif the "rivalry" effect dominates the "husband" effect.

Turning to first wives, I get :

$$n_1^* = \lambda_0.S + \lambda_1^*(\lambda_0).(T_1 - S) = \frac{n_w^{id} + \theta_1^h n_h^{id} + \theta_1^n n_1^{nat}}{1 + \theta_1^h + \theta_1^n} + n_2.\frac{\epsilon_1 - \theta_1^h}{1 + \theta_1^h + \theta_1^n}$$
$$= n_1^{NS} + n_2.B_1$$

3.2.2 Second stage : Nash equilibrium

I solve the following system :

$$n_1^* = n_1^{NS} + n_2^*.B_1$$
$$n_2^* = n_2^{NS} + n_1^*.B_2$$

I get :

$$n_i^* = (n_i^{NS} + n_{-i}^{NS}.B_i) \times \frac{1}{1 - B_1.B_2} \text{ for } i = 1,2 \tag{4}$$

I further impose that $\epsilon_i \in [0,1]$ so that $B_i \in]-1,1]$ and $(1 - B_1.B_2) \geq 0$. If $(1 - B_1.B_2) = 0$, there is no equilibrium. It happens when $\epsilon_i = 1, \theta_i^h = \theta_i^n = 0$ for $i = 1,2$, meaning that the rivalry effect is not offset by any kind of marital or biological constraint. The number of children of both wives is pushed to infinity. Another extreme case is when $B_i \to -1$ for $i = 1,2$. It happens when $\epsilon_i = \theta_i^n = 0$ and θ_i^h is very large, meaning that only the deviation costs from husband's preferences are driving fertility choices. Here, there is an infinite number of equilibria : (n_1, n_2) s.t. $n_1 + n_2 = n_h^{id}$. Both cases are easily ruled out by the fact that fertility choices are not free from any biological constraints, so θ_i^n is never equal to zero.

Note that the equilibrium of the static game is fully determined by n_1^{NS}, n_2^{NS}, B_1 and B_2. Whatever λ_0, first wives adjust their birth rate after the shock; they choose $\lambda_1^*(\lambda_0)$ such that $\lambda_0 S + \lambda_1^*(\lambda_0).(T_1 - S) = n_1^* = (n_1^{NS} + n_2^{NS}.B_1) \times \frac{1}{1 - B_1.B_2}$.

3.2.3 Third stage : back to $t = 0$

At $t = 0$, first wives maximize over λ_0 :

$$(1 - \pi) \times u(\lambda_0.T_1, 0) + \pi \times \mathbb{E}[u(n_1^*, n_2^*)]$$

Since $u(n_1^*, n_2^*)$ does not depend on λ_0, the maximization problem boils down to the problem in monogamous societies. The optimal initial birth rate is :

$$\lambda_0^* = \frac{n_w^{id} + \theta^h n_h^{id} + \theta^n \lambda_m^{nat}.T_1}{(1 + \theta^h + \theta^n).T_1} = \frac{n_0^{NS}}{T_1}$$

One testable implication is that π does not influence the optimal initial birth rate. Women with different beliefs should take the same decision in the first period. If one assumes that women have accurate private information on their probability to end up in a polygamous union, then π should be higher for women who will eventually face a rival than for women who are still the sole wife at an advanced age. One way to test the model is to compare the birth rates of first wives before the second marriage to the birth rates of relatively old women whose husband is still monogamous. If the model is valid, they should be the same. [21]

Prediction 2 *Controlling for marriage duration and preferences, birth spacing should be the same for monogamous and first wives* before *the shock.*

We can prove that an equilibrium exists as soon as $B_i \geq 0$ for $i = 1, 2$. If we come back to the Nash equilibrium described above, it exists iif $\lambda_i^* \geq 0$ for $i = 1, 2$. For the second wife, it is the case when $B_2 \geq 0$. For the first wife, it is the case when $n_1^*(S) \geq \lambda_0^*.S$. Let me note $f(S) = n_1^*(S) - \lambda_0^*.S$. $f(S)$ is monotonic, and $f(0)$ and $f(T_1)$ are both non-negative, so $f(S) \geq 0$ for all S. In other words, whatever the timing of the second marriage, the first

21. An alternative explanation for similar birth rates is that all women have the same expectation, and the shock materializes only for some of them.

wife always wants more children than she currently has at the time of the shock.

Non-cooperative models generally predict that household members are unable to reach an optimal allocation in terms of total welfare. In the Appendix, I compare the equilibrium and the outcome maximizing total welfare. I find that the total number of children would be closer to the husband's preferences if wives were to agree on maximizing total welfare instead of maximizing their own utility. The relative number of children would reflect more the difference in wives' preferences and less the difference in wives' natural fertility. I also investigate whether there are too many or too few children at equilibrium compared to the welfare-maximizing outcome. The answer depends on the relative values of $(n_1^{id} + n_2^{id})$, n_h^{id} and $(n_1^{nat} + n_2^{nat})$. In general, the non-cooperative model leads to a surplus of children ; a deficit of children would be observed only when n_h^{id} is uncommonly high.

3.3 Comparative statics

Using the closed forms mentioned above, I am able to predict how the equilibrium number of children and birth spacing should evolve with the timing of the second marriage.

Starting with first wives, the key variables characterizing the timing are the duration of their own reproductive period (T_1), the duration of the monogamous period (S) and the duration of their co-wife's reproductive period (T_2). Using the closed form for n_1^* in equation 4, I find that $\frac{\partial n_1^*}{\partial T_1} > 0$. Also, $\frac{\partial n_1^*}{\partial S} > 0$ because the natural birth rate is higher during the monogamous period. More interestingly, $\frac{\partial n_1^*}{\partial T_2}$ has the same sign as B_1. It means that an increase in the junior wife's reproductive period raises the first wife's number of children iif the "rivalry" effect dominates the "husband" effect.

Prediction 3 *The number of children of first wives should increase with their own reproductive period and with the duration of the monogamous period. It should also increase with the second wife's reproductive period iif the strategic effect is positive.*

The existence and direction of a strategic effect may also be inferred from birth spacing. One quantity of interest is the difference in optimal birth rates before and after the shock. We have :

$$\lambda_1^* - \lambda_0^* = \frac{n_1^* - n_0^{NS}}{T_1 - S} = (\frac{n_2^*.B_1}{T_1 - S} - \frac{\theta_1^n}{1 + \theta_1^h + \theta_1^n} \times (\lambda_m^{nat} - \lambda_p^{nat})) \tag{5}$$

The impact of the shock depends on the sign of the strategic effect and on the change in natural fertility. If $B_1 < 0$, birth spacing is always longer after the shock. If $B_1 > 0$, the shock leads to a lengthening of birth spacing iif the strategic effect is dominated by the change in natural fertility. The advantage of looking at the change in birth rate, rather than the level of λ_1^*, is to get rid of time-invariant unobservable characteristics of spouses, starting with their preferences.

Prediction 4 *If first wives shorten birth spacing after the shock, it must be the case that* $B_1 > 0$. *If they lengthen birth spacing, the strategic effect can be positive or negative.*

Using Equation 5, I further find that $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_2}$ has the same sign as B_1. In the case of a negative strategic effect, birth spacing should lengthen more when T_2 is large. In the case of a positive effect, it should lengthen less (if $\lambda_1^* - \lambda_0^* < 0$) or increase more (if $\lambda_1^* - \lambda_0^* > 0$) when T_2 is large.

Prediction 5 *If first wives shorten birth spacing after the shock, the reduction must be stronger when the reproductive period of the second wife is long. In the case that first wives lengthen birth spacing after the shock : they should lengthen less when the reproductive period of the second wife is long iif the strategic effect is positive.*

The signs of the derivatives with respect to S and T_1 are ambiguous. I can only derive the following implication : if $B_1 > 0$ and $B_2 > 0$, then $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial T_1} < 0$ and $\frac{\partial \lambda_1^* - \lambda_0^*}{\partial S} > 0$.

Prediction 6 *If $B_1 > 0$ and $B_2 > 0$, (i) in the case that first wives shorten birth spacing after the shock : the reduction must be stronger when they have spent more time as a monogamous wife and when their own reproductive period is shorter ; (ii) in the case that first wives lengthen birth spacing after the shock : they should lengthen less when they have spent more time as a monogamous wife and when their own reproductive period is shorter.*

Turning to second wives, the key variables characterizing the timing are the duration of their own reproductive period (T_2) and the duration of the first wife's reproductive period split into the monogamous period (S) and the bigamous one ($T_1 - S$). From equation 4, I derive that $\frac{\partial n_2^*}{\partial T_2} > 0$ and that $\frac{\partial n_2^*}{\partial (T_1 - S)}$ and $\frac{\partial n_2^*}{\partial S}$ have the same sign as B_2.

Prediction 7 *The number of children of second wives should increase with the duration of their own reproductive period. The duration of the monogamous period and the time left before the end of the first wife's reproductive period should impact fertility in the same direction. They should raise the junior wife's completed fertility iif the strategic effect is positive.*

As for birth spacing, the predictions regarding the impact of S and $(T_1 - S)$ on completed fertility still hold because $\lambda_2^* = \frac{n_2^*}{T_2}$. Again, the impact of the own reproductive period is ambiguous. I only know that $\frac{\partial \lambda_2^*}{\partial T_2} < 0$ if $B_2 > 0$.

Prediction 8 *The duration of the monogamous period and the time left before the end of the first wife's reproductive period should impact birth spacing of second wives in the same direction. They should shorten birth spacing iif the strategic effect is positive.*

Prediction 9 *If $B_2 > 0$, second wives should lengthen birth spacing when their own reproductive period is longer.*

3.4 Extensions

3.4.1 Incomplete information

When I model the interaction between both wives as a non-cooperative, simultaneous game, I implicitly assume that each player knows the payoff of the other player. In my context, it means that each wife knows the ideal number of children of the other. From the second wife perspective, it might seem reasonable to assume that she infers n_{w1}^{id} from the behavior of the first wife in the first period. She observes S and the number of children as she enters the household, so she is able to deduce the optimal initial birth rate of the first wife, and hence her preferences. Things are not as straightforward from the first wife perspective, who only observes T_2, but has no piece of evidence to infer n_{w2}^{id}.

In this extension, I consider that n_{w2}^{id} is private information of the second wife. To simplify the notations, let me call $t = n_{w2}^{id}$ the type of the second wife, and $f(t)$ the density, defined on an interval $I \in \mathbb{R}^+$. The first wife knows the distribution of types in the population of second wives.[22] I denote $n_2(t)$ the strategy played by a second wife of type t. From section 3.2.2, the best response of a second wife of type t when the first wife plays n_1 is :

$$n_2(t) = n_2^{NS}(t) + B_2.n_1 \tag{6}$$

Where $n_2^{NS}(t) = \frac{t+\theta_2^h n_h^{id}+\theta_2^n n_2^{nat}}{1+\theta_2^h+\theta_2^n}$.

What is the best response of first wives when second wives of type t play $n_2(t)$? First wives maximize their expected utility :

$$\mathbb{E}[u(n_1, n_2(t)] = \int_I u(n_1, n_2(t))f(t)dt$$

22. Note that the first wife does not update her beliefs during the game.

When $n_2(t)$ is considered as given, $\frac{\partial u}{\partial n_1}$ is linear in n_1 and in $n_2(t)$, so the FOC gives :

$$n_1 = n_1^{NS} + B_1 \int_I n_2(t) f(t) dt \tag{7}$$

The Nash equilibrium is the intersection of all best responses. Plugging the expression of $n_2(t)$ from equation 6 into equation 7, I get :

$$n_1^* = (n_1^{NS} + B_1 \mathbb{E}[n_2^{NS}]) \times \frac{1}{1 - B_1.B_2}$$

$$n_2^*(t) = (n_2^{NS}(t) + B_1.B_2(\mathbb{E}[n_2^{NS}] - n_2^{NS}(t)) + B_2.n_1^{NS}) \times \frac{1}{1 - B_1.B_2}$$

Where $\mathbb{E}[n_2^{NS}] = \frac{\int_I tf(t)dt + \theta_2^h n_h^{id} + \theta_2^n n_2^{nat}}{1+\theta_2^h+\theta_2^n}$.

Under incomplete information, the model predicts that n_1^* does not depend on n_{w2}^{id}. Since the first wife ignores the preferences of her rival, she responds to the preferences of the average rival. The equilibrium number of children, compared to the complete information case, depends on the sign of $(\mathbb{E}[n_{w2}^{id}] - n_{w2}^{id})$. When $B_i \geq 0$ for $i = 1, 2$, both wives have more children than under complete information if the ideal number of children of the second wife is below average. They have fewer children if the number is above average. This framework can be easily extended to relax the assumption that n_{w1}^{id} is known by the second wife.

3.4.2 Sequential game

So far, I have considered that either the first wife always remains in a monogamous union, or a second wife arrives at date S and both wives play a simultaneous game. In fact, when the first wife is relatively old as the second marriage takes place, the game is not simultaneous, but sequential. Indeed, the first wife has already given birth to $n_1 = \lambda_0.T_1$ children, and she can no longer update this quantity. Then the second wife chooses her best response to n_1, and payoffs are paid when the reproductive period of the second wife is over. Therefore, if $T_1 \leq S$, the interaction between both wives is best described by a Stackelberg leadership model.[23]

At $t = 0$, first wives consider three scenarios : (i) $S = +\infty$ or $T_2 = 0$: no strategic interaction ; (ii) $S \geq T_1$ and $T_2 > 0$: sequential game ; and (iii) $S < T_1$ and $T_2 > 0$: simultaneous game. They split π, the probability that a fertile second wive arrives, into π_a the probability that she arrives when they are too old to adjust their own fertility, and π_b the probability that she arrives when they are young enough. Keeping the notations of section 3.2.2, first wives maximize their expected utility over λ_0 :

$$(1-\pi) \times u(\lambda_0.T_1, 0) + \pi_a \times \mathbb{E}[u(\lambda_0.T_1, n_2(\lambda_0.T_1))] + \pi_b \times \mathbb{E}[u(n_1^*, n_2^*)]$$

Where $n_2(\lambda_0.T_1)$ is the best response of the second wife when she faces a first wife with $\lambda_0.T_1$ children. As already noted above, $u(n_1^*, n_2^*)$ does not depend on λ_0.

To find the subgame perfect Nash equilibrium, I solve the game by backward induction. I start by considering the last stage of the sequential game. Building on section 3.2.2, I know that $n_2(n_1) = n_2^{NS} + B_2.n_1$. First wives anticipate the reaction of second wives. Let me compute the best strategy of first wives for a given T_2 (which determines n_2^{NS}). They maximize $u(n_1, n_2^{NS}(T_2) + B_2.n_1)$. The FOC gives :

$$n_1^{st}(T_2) = \frac{n_w^{id}(1-B_2\epsilon_1) + \theta_1^h n_h^{id}(1+B_2) + \theta_1^n n_0^{nat} + n_2^{NS}(T_2)(\epsilon_1(1-B_2\epsilon_1) - \theta_1^h(1+B_2))}{(1-B_2\epsilon_1)^2 + \theta_1^h(1+B_2)^2 + \theta_1^n} \quad (8)$$

Note that n_1^{st} increases with T_2 iif $B_1^{st} = \epsilon_1(1-B_2\epsilon_1) - \theta_1^h(1+B_2) \geq 0$. In the simultaneous game, I found that n_1^* increases with T_2 iif $\epsilon_1 - \theta_1^h \geq 0$. To understand the difference, one needs to consider the cross-derivative of $u_1(.)$ with respect to n_1 and n_2. When n_2 is taken

23. In the rest of the section, I use the superscripts *st* to denote the equilibrium quantities when I introduce the Stackelberg interaction in the basic model.

as exogenous, I have :

$$\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\epsilon_1 - \theta_1^h)$$

When n_2 depends on n_1 in such a way that $\frac{\partial n_2}{\partial n_1} = B_2$, the expression is :

$$\frac{\partial^2 u_1}{\partial n_1 \partial n_2} = 2(\epsilon_1 \times (1 - \epsilon_1 B_2) - \theta_1^h \times (1 + B_2))$$

When n_2 is fixed, having one more child for the first wife means that her relative number of children increases by one unit, and that the total number of children of the husband increases by one unit. Whereas when n_2 depends on n_1, having one more child for the first wife means that the second wife will have B_2 additional children. So her relative number of children increases by $(1 - \epsilon_1 B_2)$, and the total number of children of the husband increases by $(1 + B_2)$.[24]

Now, first wives do not know T_2 before the second marriage takes place. But they have some information about the timing of events. They first know that in the sequential scenario, $S \geq T_1$ and $T_2 > 0$. Moreover, T_2 is bounded above by $T_h - S$, where T_h is the length of the husband's reproductive period at $t = 0$. Therefore, the distribution of T_2 depends on S : the later the second marriage takes place in the husband's life, the shorter the time left to the second wife to have children. I denote $g(T_2|S)$ the conditional distribution of T_2 given S, and $h(S)$ the distribution of S on the interval $[T_1, T_h]$. First wives maximize their expected

24. Let me rewrite $B_1^{st} = B_1(1+\theta_1^h+\theta_1^n) - B_2(\theta_1^h+\epsilon_1^2)$. It may be the case that $B_1^{st} < 0$ even if $B_1 \geq 0$, for instance when B_2 is much larger than B_1. The intuition is that, when the second wife reacts very strongly to an increase in n_1, the increase in the relative number is small compared to the increase in the total number of children. On the other hand, when $B_1 \geq B_2 \geq 0$, then $B_1^{st} \geq 0$. In other words, when the "strategic" reaction of the second wife is not stronger than the one of the first wife, the first wife is always better off raising her number of children when she faces a more fertile rival.

utility on n_1 :

$$\mathbb{E}[u(n_1, n_2^{NS}(T_2) + B_2.n_1)] = \int_{S=T_1}^{T_h} \int_{T_2=0}^{T_h - S} u(n_1, n_2^{NS}(T_2) + B_2.n_1) g(T_2|S) h(S) dT_2 dS$$

The derivative of $u(.)$ is linear in T_2 so I can write the equilibrium number of children using equation 8 :

$$n_1^{st} = n_1^{st}(\mathbb{E}[T_2|S \in [T_1, T_h]])$$

Under standard density functions for $g(.)$ and $h(.)$, $\mathbb{E}[T_2|S \in [T_1, T_h]]$ is increasing in $(T_h - T_1)$.[25] So n_1^{st} is also increasing in $(T_h - T_1)$ as long as $B_1^{st} \geq 0$.

Let me come back to $t = 0$ and consider the optimal initial birth rate under the three scenarios that I mentioned below : (i) no strategic interaction : $\lambda_0 = \frac{n_0^{NS}}{T_1}$; (ii) sequential game : $\lambda_0 = \frac{n_1^{st}}{T_1}$; and (iii) simultaneous game : indifferent between any $\lambda_0 \geq 0$. The first-order condition is a weighted average of the first-order condition under no strategic interaction (weight $(1 - \pi)$) and the first-order condition of the sequential game (weight π_a). As result, the optimal initial birth rate λ_0^{st} lies between $\frac{n_0^{NS}}{T_1}$ and $\frac{n_1^{st}}{T_1}$.

How does n_1^{st} compare to n_0^{NS} ? It depends on the sign of B_1^{st} and on the relative magnitude of n_{w1}^{id} and n_h^{id}. The case that seems the most consistent with empirical evidence is $B_1 \approx B_2 \geq 0$ (implying that $B_1^{st} \geq 0$) and $n_{w1}^{id} \leq n_h^{id}$. In this case, $n_1^{st} \geq n_0^{NS}$. When the strategic reaction is similar for both wives, and the husband wants more children than the first wife, the likelihood of a sequential game raises the initial birth rate. The first wife intensifies her fertility to improve her position in the event of a late second marriage.

I further test for such a strategic overshooting by comparing the choices of women more or less exposed to the risk of a late second marriage. The idea is to exploit the variation

25. For instance, if the distribution of S is uniform on $[T_1, T_h]$ and the distribution of T_2 conditional on S is uniform on $[0, T_h - S]$, then $\mathbb{E}[T_2|S \in [T_1, T_h]] = \frac{T_h - T_1}{4}$.

in $(T_h - T_1)$, which is driven by the age difference between the first wife and the husband. When they have a very large age difference, then $(T_h - T_1) = 0$ because the length of the reproductive period of the couple is determined by the length of the husband's period. It is very unlikely that the husband starts having children with another wife when the first one is already in her late forties because he will either be dead or infertile. On the contrary, when the age difference is low, then $(T_h - T_1)$ may be as large as 15 or 20 years, which leaves time for a potential rival to have many children.

Prediction 10 *If $B_1 \approx B_2 \geq 0$ and $n_{w1}^{id} \leq n_h^{id}$, then the number of children of monogamous wives should be higher if the age difference with the husband is lower. Also, birth spacing should be shorter.*

4 Empirical Tests

In this section I test empirically the predictions about completed fertility and birth spacing. I use PSF data because information on the timing of events is very detailed. On the other hand, there is no information on preferences, so I control for the predictors of the ideal number of children identified in Table A.2. I also include characteristics of husband's occupation (income and a dummy for the public sector) which were not available in DHS data but are likely to influence preferences. Moreover, I control for the co-residence status, having children from previous unions and having dead children from current union. To mitigate the issue of infertile unions, I exclude women having no child with the current husband in all tests.

4.1 Testing the predictions on completed fertility

To test the predictions on the total number of children, I consider the sub-sample of women over 45 years old, who have reached the end of their reproductive life, in order to avoid censoring issues. Starting with first wives, the model predicts that the timing of the second marriage should have an impact on their number of children. In line with Prediction 3, the number of children of first wives increases with the duration of their own reproductive period and with the duration of the monogamous period. Coefficients in Table 7 are of expected sign, although the estimation is sometimes imprecise due to the small sample size. As for the impact of the co-wife's reproductive period, it is clearly is positive. I define a dummy for co-wives with a long reproductive period, and another for co-wives with a short reproductive period. First wives eventually have more children when the second wife has a long time left before fertility decline, and fewer children when that time period is short. The magnitude and significance of the effect is larger when I restrict the sample to unions with exactly two competitors in order to match more closely to setting of the model. These results imply that the strategic effect is positive : first wives react to more births by the co-wife by having more children themselves.

Turning to second wives, even if the sample of women over 45 years old is small, I am able to test the prediction on the number of children in Table 8. The reproductive period has a positive impact on completed fertility. Moreover, second wives have fewer children when the monogamous period was short, and when the first wife has no time left to react. Both effects point to the existence of a positive strategic reaction.

Looking at completed fertility gives a first hint that B_1 and B_2 are both positive. However, the small sample sizes prevent me to take a definitive stance. To mitigate the lack of power, I turn to birth spacing. Shifting the unit of observation from woman to birth substantially expands the sample size.

4.2 Testing the predictions on birth spacing

To test the predictions on birth spacing, I consider the sub-sample of women below 45 years old, for whom I know the complete birth history. One issue is that I do not observe the birthdate of deceased children, which might lead me to overestimate the true duration between all successive births. The best I can do is including a dummy for deceased children as a control. I estimate a duration model of birth intervals to deal properly with censoring, using the Cox method of estimation. My variable of interest is the duration between successive births, measured in months. I consider intervals between births i and $(i + 1)$ for $i = 0$ (marriage) to $i = 11$ (highest parity observed in the sample), and run a pooled regression of all intervals. I include a dummy for each rank of birth, in order to allow different baseline hazard across parities, and I control for mother's age and age squared at each birth. Then, I use robust standard errors clustered at the woman level to account for the correlation between the error terms related to the different birth intervals of the same woman.

I start with an important prediction to test the validity of the model : senior wives are predicted to choose the same birth spacing as monogamous wives before the shock (Prediction 2). In Table 9, I show that first wives do behave like monogamous wives as long as the second wife has not arrived. I restrict the sample to women over 40 to ensure that most women in the reference category "monogamous wives" will remain the sole wife in this union. I only consider births by wives who were monogamous as they gave birth ; then I construct a dummy "Future first wives" indicating whether the mother is in a polygamous union at the time of the survey. Controlling specifically for marriage duration and predictors of preferences, I find that the future status has no impact on past durations.

The second test is about the impact of the shock on first wives' birth spacing (Prediction 4). In this specification, baseline hazards are specific to each woman and I estimate the impact of a change in union status, from monogamous to polygamous.[26] The advantage is that all time-invariant characteristics are included in the individual baseline hazard, which greatly mitigates the omitted variable issue. I only have to control for birth-specific variables such as rank and mother's age. Table 10 shows that birth spacing lengthens after the shock.

26. I use a stratified partial likelihood to get rid of the individual baseline hazards.

I create a dummy "After the shock" equal to one if the birth occurred once the second wife has arrived. I estimate the impact of the shock on the very next birth interval (columns 1 and 3) and all subsequent birth intervals (columns 2 and 4). In the last columns, I include monogamous wives in the sample to improve the precision of the estimates.[27] Hazard ratios are always lower than one, meaning that durations are longer once the second wife has arrived. As explained in the model, a lengthening of birth spacing can be consistent with either positive or negative strategic effects.

To confirm that strategic effects are positive, I examine if the change in birth rate varies with the timing of the second marriage. Theoretically, it is driven by (i) the strategic reaction, which is proportional to n_2^*, and (ii) the variation in natural fertility, which is proportional to $(T_1 - S)$. From the previous test, I know that first wives lengthen birth spacing after the shock. In this case, the model predicts that, if B_1 and B_2 are both positive, first wives should lengthen less when T_2 and S are longer, and lengthen more when T_1 is longer (Predictions 5 and 6). Using the previous specification with individual baseline hazards, I interact the dummy "After the shock" with the timing variables. Coefficients reported in Table 11 are always of expected signs. The effect of T_2 is precisely and robustly estimated : the difference in birth rate is less and less negative as T_2 increases, because the equilibrium number of rivals rises. S has a similar effect, because an increase in S (holding T_1 constant) reduces the time spent in polygamous union on the one hand, and raises the number of rivals on the other hand. Last, the difference in birth rate is more and more negative as T_1 increases. The intuition is that the rise in time spent under the polygamous regime more than compensates the rise in the number of rivals. Indeed, the effect of T_1 on n_2^* is only second-order since it goes through n_1^{NS}. The same holds for S. This may explain why the coefficients on the interaction terms with T_1 and with S are not always significant.

27. They do not contribute to estimating the impact of the shock, because their union status never changes across births, but they help with birth-specific controls.

Going back to equation 5, these results imply that $\lambda_1^* - \lambda_0^* < 0$ while $B_1 > 0$. The overall impact of the shock is negative because the change in natural fertility is large enough to dominate the positive strategic effect. Since the strategic effect is proportional to n_2^*, it is only possible when n_2^* is not too large, more precisely when $n_2^* < \frac{\theta^n.(\lambda_m^{nat} - \lambda_p^{nat}).(T_1 - S)}{\epsilon_1 - \theta_h^1}$. I need to make further assumptions on the values of the parameters to assess whether this condition is likely to hold in the majority of households. I use estimations on monogamous unions for $\theta^h \approx 1/2$, $\theta^n \approx 3$ and $\lambda_m^{nat} \approx 1/3$. Under the additional assumption that $\lambda_p^{nat} = \frac{\lambda_m^{nat}}{2}$ and $\epsilon = 1$, the condition rewrites $n_2^* < (T_1 - S)$. This is verified in 90% of households. [28] Thus, my framework brings together the result of demographers that, on average, fertility is lower in polygamous unions, and the claim by anthropologists and sociologists that the reproductive rivalry between co-wives is strong.

The last prediction deals with the impact of the timing on second wives' birth spacing. To be consistent with previous results, I should observe that birth intervals are shorter when S and $(T_1 - S)$ are high, and longer when T_2 is high (Predictions 8 and 9). When I know the complete birth history of the first wife, I can deduce n_{ini}, the number of children she had at the time of the shock. Prediction 8 would be modified to state that birth intervals of second wives are shorter when n_{ini} is higher, holding $(T_1 - S)$ constant. n_{ini} is much more informative than S because it captures the optimal birth rate of the first wife. [29] Table 12 summarizes the test using S in column 1 and n_{ini} in column 2. I return to a specification with a baseline hazard common to all women, because the covariates of interest do not vary across births for a given woman. Again, signs are in line with expectations. The hazard rate is increasing in all predictors of the first wife's completed fertility : $(T_1 - S)$, S and n_{ini}. As predicted, it is decreasing in T_2. If I break down the effects by birth ranks, they are

28. The distribution of the second wife's final number of children (excluding infertile unions) is $Q_1 = 2$, $Q_2 = 4$ and $Q_3 = 6$ while the distribution of $T_1 - S$ in the same sample is $Q_1 = 9$, $Q_2 = 16$ and $Q_3 = 22$.

29. Empirically, the duration of the monogamous period is a strong predictor of the number of children already born at the time of the shock. If I regress n_{ini} on S and no constant, I find a coefficient of 0.24 highly significant. It corresponds to one birth every four years on average, or five children in 20 years of marriage.

particularly strong on the duration between marriage and first birth.

To sum up, all empirical tests support the existence of strategic interactions in polygamous unions. Children are strategic complements : characteristics raising the fertility of one wife affect positively the fertility of her co-wife.

4.3 More empirical evidence of strategic interactions

4.3.1 Tests derived from the sequential game

In the extension transforming the model in a sequential game, I predict a strategic overshooting of wives in monogamous unions exposed to the risk of a late second marriage. Prediction 10 states that fertility should be more intense when $(T_h - T_1)$ is higher, meaning that a potential rival would enjoy more time to have children. It is crucial in this test to control for the preferences of each spouse because their age difference is correlated to their ideal family size. On the other hand, I do not need information on the timing of unions since I consider anticipations. For these reasons, I perform the test on DHS instead of PSF.

Regarding completed fertility, the prediction is verified. Table 13 shows that, controlling for T_1 and the preferences of each spouse, $(T_h - T_1)$ has a positive and significant impact on the final number of children. Then I investigate whether the effect is truly linear or driven by the difference between women not exposed at all and the others. I create three categories of women depending on their exposure to the risk of a late second marriage : not exposed if $(T_h - T_1) = 0$, weakly exposed if $(T_h - T_1)$ is below the median ; strongly exposed if $(T_h - T_1)$ is above the median. I find that women not exposed have significantly fewer children than the others. Also, among exposed women, the degree of exposure matters : strongly exposed women have more children than weakly exposed ones.

Tests on birth spacing are reported in Table 14. The effect of $(T_h - T_1)$ is of predicted sign, although not significant. In the second column, I exclude durations between marriage and first birth because it may take more or less time for the couple to consummate the marriage depending on the age difference between spouses. Such a mechanism might confound the result of my test. In the third column, I interact $(T_h - T_1)$ with different ranks of birth ; the impact is all the larger as the rank is high, and it is significant after birth 5. One interpretation is that wives update upwards the relative likelihood of a late marriage as time passes by. [30]

4.3.2 Tests based on the gender composition of children

So far, I have considered only the quantity of children, putting quality aside. Nonetheless, it might be argued that co-wife rivalry is also about children's characteristics such as, for instance, educational achievement, social success, commitment to norms or responsibility taken in the family welfare. According to the literature on Africa, one characteristic plays a key role : gender. Having sons substantially improves women's status and security (Lesthaeghe 1989). It is particularly true in patrilineal ethnic groups, and where the influence of the Islamic law is strong, like in Senegal where 95% of the population is Muslim. In a previous work on Senegal, I show that women have a stronger preference for sons when the current husband already has children from ex-wives, whether divorced or deceased. The explanation rests on the rivalry for inheritance between the husband's children. In presence of children from ex-wives, current wives need a son to secure access to their late husbands' resources in case of widowhood (Lambert and Rossi 2014). The same rationale might be at play in polygamous households, and even exacerbated by the rivalry for *current* resources, be it material or emotional.

The hypothesis I want to test in this section is that the gender of children matters in polygamous households. Ideally, I would like to predict how the birth of boy vs. a girl

30. In the model, to keep things simple, I assume that the ratio $\frac{\pi_a}{1-\pi}$ remains constant over the first period. The idea is that, even if the probability of an early second marriage (π_b) decreases as time goes by, it does not change the relative likelihood of a late marriage compared to the likelihood of no second marriage. In fact, first wives seem to consider that $(1-\pi)$ is fixed, and that the decreasing risk of an early second marriage is fully converted into a rising risk of late marriage.

impacts the subsequent optimal birth rate of both wives, and whether this effect depends on the gender composition of the other wife. However, my model of fertility choices lacks a true time dimension to adequately account for the uncertainty related to a child's gender. Therefore, I build on the above-mentioned work on the rivalry with ex-wives to derive the following predictions regarding co-wives. On the one hand, the arrival of a second wife should exacerbate the preference for sons of first wives. Indeed, they move from a situation in which no other child can compete with their own offspring, to a situation with rivals. So the necessity to have a son should be stronger after the shock. On the other hand, the behavior of the second wife should depend on the gender composition of the first wife's children, boys representing a more serious threat than girls. The second wife's fertility should therefore increase more with the number of boys than with the number of girls already born to the first wife.

To test the prediction on the change in son preference of first wives, I consider a similar specification as the one dealing with the impact of the shock. I estimate a duration model with a baseline hazard specific to each woman. I introduce a dummy "No son" equal to one if the woman had still no son at the time of the index birth. This variable varies across births and captures the impact of having only daughters vs. at least one son on the next interval, holding the birth rank constant. If the hazard ratio is larger than one, meaning that having only daughters decreases the expected interval, then one can infer the existence of son preference.[31] Then I interact the gender composition with the dummy "After the shock" to test whether the arrival of the second wife has an impact on son preference. Results for unions with exactly two competitors are reported in Table 15. Son preference exists before the shock : the hazard ratio on "No son" is significantly greater than one. But it is substantially exacerbated by the second marriage : the hazard ratio on the interaction term, capturing

31. In Rossi and Rouanet (2014) I discuss in more details how to infer the existence of gender preferences using duration models of birth intervals.

the difference in son preference before and after the shock, is equal to two. Controlling for birth ranks and mother's age at birth, I find that the same woman displays much larger son preference once her rival has arrived than she used to do.

Regarding the prediction on second wives, I start from the specification used in the second column of Table 12. I split n_{ini}, the first wife's number of children at the time of the shock, into $Boys_{ini}$, her number of boys and $Girls_{ini}$, her number of girls. I restrict the analysis to unions with exactly two competitors. Table 16 shows that second wives react more to the number of boys than to the number of girls. Coefficients on $Boys_{ini}$ and $Girls_{ini}$ are both positive, but the former is 60% larger than the latter and only $Boys_{ini}$ has a significant impact.

Both tests suggest that co-wife rivalry raises the relative value of sons against daughters. It is an additional piece of evidence that potential and realized fertility outcomes of one wife influence the behavior of the other wife.

5 Discussion

For a long time, controlling population growth has been a concern for policy makers. The general view is that population becomes a problem when it starts growing faster than resources, because such a dynamic leads to general impoverishment. [32] In recent years, the rising environmental awareness has brought the issue back up (Sachs 2008). Since costs and benefits to children are partly passed on society, there is a role for public policies to influence fertility decisions made by parents. Health concerns are another rationale for state intervention, insofar as frequent pregnancies raise the risk of maternal and child mortality. In Africa, more and more effort has been devoted to reducing the rate of population growth. In 1976, policies to lower the level of fertility were implemented in only 25% of African countries,

32. For an overview of the debates over the so-called "population problem", see Dasgupta (1995).

while that proportion increased up to 83% in 2013 (United Nations 2013). Measures are generally taken on the supply-side to increase access to family planning services and to birth control methods. However, identifying the determinants of the demand for children is crucial to design adequate policies. By providing a simple framework to analyze which forces drive fertility, this paper shows that policy makers should definitely take into account the structure of households.

Policy recommendations to curb fertility have generally been designed for monogamous societies. They build on the fact that, on average, wives want fewer children than their husband, and fewer children than they would have in a natural fertility regime. For instance, Sen (1999) argues that the main drivers of fertility transition are improving women's bargaining power and facilitating access to birth control methods. In my framework, it means alleviating the marital and natural constraints weighing on monogamous women. Indeed, the lower θ_h and θ_n, the closer n^* to n_w^{id}, and therefore, in the vast majority of households, the lower n^*.

However, these standard recommendations might be counterproductive in polygamous societies. The model presented in this paper states that fertility choices in polygamous unions differ from choices in monogamous unions along three dimensions : (i) an overbidding effect, (ii) a substitution effect and (iii) an exposure effect. The overbidding effect reflects the reproductive rivalry between co-wives and drives fertility upwards. The substitution effect comes from the fact that all children are perfect substitutes from the husband's point of view, which limits the fertility of each wife. Last, the intensity of exposure to the risk of pregnancy is lower in polygamous unions, which creates a downward exposure effect.

The three effects are easy to distinguish in equation 5. The difference in first wives' birth rate before and after the shock has the same sign as $n_2^*.\epsilon - n_2^*.\theta^h - \theta^n.(\lambda_m^{nat} - \lambda_p^{nat})(T_1 - S)$. The term $n_2^*.\epsilon$ captures the overbidding effect, $n_2^*.\theta^h$ reflects the substitution effect, and $\theta^n.(\lambda_m^{nat} - \lambda_p^{nat})(T_1 - S)$ is the exposure effect. Observing the same woman in both types of union makes it possible to estimate an order of magnitude of each effect. According to empirical tests carried out in this paper, first wives tend to lengthen birth spacing after the shock. It means that the overbidding effect is dominated by the sum of substitution effect and exposure effect. On the other hand, I find that the strategic reaction is positive, meaning that the overbidding effect dominates the substitution effect.

So, overall in Senegal, polygamy is associated with lower birth rates at the micro level.[33] But this might change if women's choices are less and less constrained by husbands and biology. Indeed, the negative correlation between polygamy and fertility is driven by the marital and natural constraints. If θ_h and θ_n decrease, the relative magnitude of the overbidding effect rises. It makes it more likely to observe an overall positive impact of polygamy on fertility. The correlation might also become positive if couples comply less with social norms related to conjugal obligations. In this case, the gap between λ_m^{nat} and λ_p^{nat} might be closed and the exposure effect would disappear.

A key lever to reduce fertility in polygamous societies is acting on the causes of co-wife reproductive rivalry, that is reducing ϵ. As already explained, children are women's best claim to current and future husband's resources. Thus, policies should aim at disassociating women's status and economic security in polygamous households from their relative number of children. One way ahead is to give more opportunities for self-support to women. Concretely, it means easing labor market restrictions and constraints stemming from social norms to improve female force participation. Another recommendation would be to reform inheritance practices in order to improve widows status, for instance by entitling wives to a significant share of the bequest irrespective of the number of children. More generally, any policy reducing women's reliance on their offspring is likely to curb the demand for children.

33. Note that, at the macro level, polygamy has always been associated with higher fertility (Lesthaeghe 1989). This institution is indeed closely related to early marriages and quick remarriages of women. Although the natural birth rate is lower, the length of women's exposure to marriage is maximized, so that, overall, fertility is higher in polygamous societies.

6 Conclusion

This paper proposes a new model to account for fertility choices in polygamous unions. It specifies the main drivers of a couple's fertility, namely the preferences of each spouse and the duration of the reproductive period. I further derive empirical tests for the existence of strategic interactions in bigamous households. Data from Senegal shows that children of both wives are strategic complements meaning that one wife will raise her fertility in response to an increase by her co-wife.

These results have strong implications for population policies in Africa. The general consensus is that giving women a greater say in fertility decisions and more efficient means to implement them are key drivers of the fertility transition. I claim that these standard recommendations might backlash in polygamous societies because co-wife rivalry would be less tempered by male involvement and biological constraints. To curb the demand for children, women's empowerment must be understood in a much broader sense. It is only when women's status and economic security are not determined by their relative number of children, that the fertility transition will no longer be hampered by polygamy.

Taking into account strategic interactions in the household is therefore a prerequisite to design effective population policy instruments. As already noted by Alderman, Chiappori, Haddad, Hoddinott, and Kanbur (1995), the unitary view of the household ignores or obscures important policy issues that are especially relevant in the context of developing economies. This paper is one of the few attempts to open the black box of non-nuclear households, but further research is needed to understand decisions related to most topics on the development agenda such as poverty, migration, labor supply, home production, land tenure, health and education.

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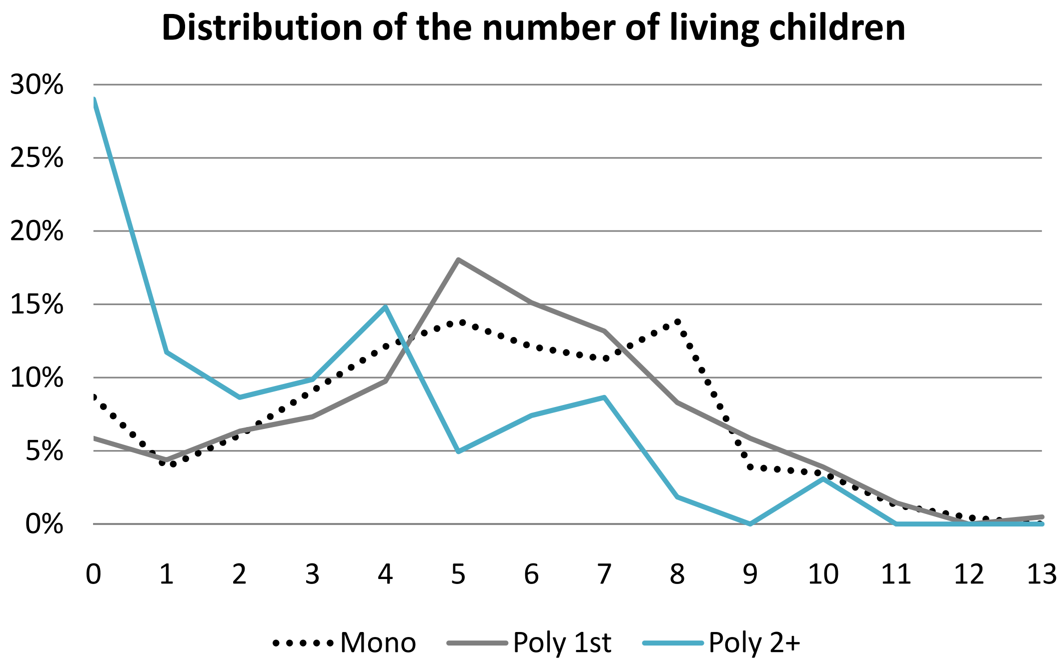
Figures and Tables

Figure 1: Number of children in current union, by union type



Data : PSF. Sample : married women over 45 years old. Mono= monogamous wives (231 obs), Poly 1^{st}= senior wives (205 obs) and Poly 2+= junior wives (162 obs).

Table 1: Impact of ages at marriage on total fertility

Threshold for large age difference	15 years	18 years
Age wife	-0.078***	-0.087***
	(0.027)	(0.022)
Age husband	-0.011	0.000
	(0.027)	(0.021)
Age wife * Large age difference	0.021	0.040
	(0.048)	(0.059)
Age husband * Large age difference	-0.049	-0.104*
	(0.044)	(0.055)
Specific controls	Union type	
Additional controls	Yes	
Nb obs	564	
F-test (p-val)		
Age wife + Age wife * Large=0	0.165	0.413
Age husband+ Age husband * Large=0	0.093*	0.045**

Data : PSF. Sample : women over 45 years old.

Dep. var. : number of children in current union.

Age means age at marriage ; Union type : monogamous, senior wife, junior wife ; Large age difference is dummy equal to 1 if the age difference between spouses is larger than 15 years in column 1 or 18 years in column 2.

Additional controls : co-residing with husband, education, area of residence, at least one child from previous union, being in first marriage, employment status, ethnic group.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 2: Number of children in current union, by union type

Sample	All		At least one child		
Polygamous union	-1.071***				
	(0.232)				
Senior wife		0.188	0.138	-0.130	-0.139
		(0.250)	(0.230)	(0.228)	(0.225)
Junior wife		-2.512***	-1.370***	-0.933***	-0.948***
		(0.260)	(0.270)	(0.279)	(0.276)
T				0.081***	0.059***
				(0.013)	(0.014)
Children from previous unions					-1.236***
					(0.312)
Constant	5.074***	5.074***	5.477***	3.687***	4.364***
	(0.183)	(0.170)	(0.157)	(0.338)	(0.373)
Observations	674	674	568	551	550

Data : PSF. Sample : women over 45 years old. In the last three columns, I restrict the sample to women having at least one child with their current husband.

Dep. Var. : number of children in current union.

T = min (45- wife's age at marriage ; 60-husband's age at marriage) : length of couple's reproductive period.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 3: Fertility in current union, by mother's rank

Dep. var.	Number of children	Birth intervals
Estimation	OLS	Cox (hazard ratios)
rank1	-0.134	0.889**
	(0.226)	(0.046)
rank2	-0.870***	0.783***
	(0.294)	(0.041)
rank3	-1.222**	0.696***
	(0.533)	(0.075)
rank4	-1.467	0.561**
	(1.065)	(0.149)
Controls	T and children from previous unions	
Observations	550	3717

Data : PSF. Sample : in column 1, women over 45 years old having at least one child with current husband. In column 2, women below 45 years old, having at least one child with current husband, and for whom the complete birth history is known.

T = min (45- wife's age at marriage ; 60-husband's age at marriage).

In column 1 : OLS estimation ; the unit of observation is the woman. In column 2 : Cox estimation on pooled durations between births i and $(i+1)$; the unit of observation is the birth ; Breslow method to handle ties among non-censored durations ; robust standard errors clustered at the woman level.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

In both regressions, the coefficient on *rank*1 is significantly different from the coefficients on all other ranks, but the coefficients on *rank*2, *rank*3, and *rank*4 are not significantly different from one another (pair-wise F-tests).

Table 4: Ideal number of children

Dep. var. Sample	n_w^{id} Wives	n_h^{id} Husbands	$n_h^{id} - n_w^{id}$ Couples
Observations	3308	3308	3308
Non-numerical answer	773	1017	1484
Numerical answer	2535	2291	1824
Mean	5.7	9.0	3.1
Std. Dev.	2.3	5.7	5.6
Q_1	4	5	0
Q_2	5	7	2
Q_3	7	10	5

Data : DHS, waves 2005 and 2010. Weights.
n_w^{id} and n_h^{id} are the ideal number of children reported by women and men, respectively. The last column shows the difference between these numbers within a couple.
"Non-numerical answer" means that the respondent answered "I don't know" or any non-numerical statement (e.g. "it is up to God") to the question "How many children would you like to have, or would you have liked to have, in your whole life ?". Statistics are computed on the sample of individuals who gave a numerical answer for the first two columns, and on the sample of couples in which *both* spouses gave a numerical answer for the last column.

Table 5: Timing of unions

Median age	Husband	1^{st} wife	2^{nd} wife
First marriage	28 (Q1=24 ; Q3=32)	17 (Q1=15 ; Q3=21)	na
Second marriage	40 (Q1=34 ; Q3=46)	29 (Q1=24 ; Q3=36)	22 (Q1=17 ; Q3=31)

Data : PSF. Sample : polygamous unions (411 unions).
If I restrict the sample to bigamous unions (321 unions), all median ages increase by one year.

TABLE 6: Estimating the model on monogamous and polygamous unions

Sample	Monogamous	Polygamous
n_w^{id}	0.232**	-0.024
	(0.092)	(0.096)
n_h^{id}	0.128**	0.067*
	(0.064)	(0.039)
T	0.205***	0.188***
	(0.046)	(0.047)
Constant	-1.083	1.785
	(0.985)	(1.302)
R^2	0.36	0.14
pval $n_w^{id} = n_h^{id}$	0.36	0.39
Observations	109	151
Structural parameters		
θ^h	0.55	na
θ^n	2.77	na
λ^{nat}	0.32	na
n^{nat} (mean)	8.30	na

Data : DHS. Sample : women over 40, in first union, at least one child. For monogamous unions, we restrict to husbands in their first union. Weights.

Dep. Var. : total number of births. T = min (45- wife's age at marriage ; 60-husband's age at marriage). $n^{nat} = \lambda^{nat} \times$ mean(T) with mean(T) = 26.

For polygamous unions, the model specification is not right. Since the coefficient on n_w^{id} is not significantly different from zero, I cannot compute the estimates of the structural parameters.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 7: Testing the prediction on the number of children of first wives

Sample	All	All	Exactly two competitors
T_1	0.220	0.245	0.300
	(0.198)	(0.189)	(0.270)
S	0.056	0.114**	0.230***
	(0.069)	(0.048)	(0.072)
T_2	0.023		
	(0.073)		
T_2^{low}		-2.098***	-2.829***
		(0.761)	(0.999)
T_2^{high}		0.580	2.474**
		(0.684)	(1.129)
Specific controls	Predictors of preferences		
Additional controls	Yes		
Observations	101	101	67

Data : PSF. Dep. var. : number of children in current union. Sample : first wives over 45 years old who were younger than 45 years old as the second wife arrived, having at least one child from current union. In column 3, I restrict the sample to unions with exactly two competitors, meaning that I exclude infertile co-wives and unions with more than two fertile co-wives.

S = (first wife age at shock - first wife age at marriage) ; T_1 = min (45-first wife's age at marriage ; 60-husband's age at first marriage) ; T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; T_2^{low} : T_2 is below Q_1 (10 years) and T_2^{high} : T_2 is above Q_3 (22 years).

Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union.

Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

Table 8: Testing the prediction on the number of children of second wives

Sample	All	All
T_2	0.074	0.334*
	(0.244)	(0.160)
S	0.040	
	(0.107)	
$T_1 - S$	0.013	
	(0.157)	
S^{low}		-3.354**
		(1.013)
$(T_1 - S)^{low}$		-2.307
		(1.753)
Specific controls	Predictors of preferences	
Additional controls	Yes	
Observations	48	48

Data : PSF. Dep. var. : number of children in current union. Sample : second wives over 45 years old, having at least one child from current union.
T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; S = (first wife age at shock - first wife at marriage) ; S^{low} : S is below the median (9 years). $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S ; $(T_1 - S)^{low}$: $T_1 - S = 0$.
Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union.
Significance levels : * p<0.10, ** p<0.05, *** p<0.01.

TABLE 9: Comparing monogamous wives and first wives *before* the shock

Hazard ratios	$i \geq 0$	$i \geq 1$
Future first wives	0.970	0.953
	(0.148)	(0.154)
Specific controls	T and predictors of preferences	
Additional controls	Yes	
Observations	584	477

Data : PSF. Dep. var. : duration between births i and $(i+1)$. Birth 0 corresponds to the date of marriage. In column 2, I exclude durations between marriage and first birth, restricting thus the sample to women having at least one child. Sample : monogamous and senior wives *before the shock*, between 40 and 45 years old, for whom the complete birth history is known.

"Future first wives" is equal to 1 if the woman is in a polygamous union at the time of the survey.

Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth i, a dummy for each i.

Cox estimation ; Breslow method to handle ties among non-censored durations.

Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 10: Estimating the impact of the shock on first wives' birth spacing

Hazard ratios	First wives		Monogamous and first wives	
	The next interval	All intervals	The next interval	All intervals
After the shock	0.514**	0.870	0.559**	0.773
	(0.157)	(0.187)	(0.142)	(0.143)
Controls	Birth-specific variables			
Observations	480	677	2410	2607

Data : PSF. Dep. var. : duration between births i and $(i+1)$. In columns 1 and 3, I consider all intervals before the shock, and the very next interval after the shock. In columns 2 and 4, I consider all intervals. Durations between marriage and first birth are always excluded (when I include them, estimates are less significant).

Sample : women below 45 years old, for whom the complete birth history is known, having at least one child from current union ; only first wives in the first two columns, monogamous and first wives in the last two columns.

"After the shock" is a time-varying variable indicating if the second wife has arrived.

Controls : A dummy for each i, mother's age and age squared at birth i.

Cox estimation ; Stratified likelihood with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations.

Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 11: Testing the predictions on the birth spacing of first wives

Hazard ratios	First wives		Monogamous and first wives	
	The next interval	All intervals	The next interval	All intervals
After the shock	0.619	0.395	0.822	0.317
	(1.283)	(0.483)	(1.410)	(0.356)
After the shock * T_1	0.832*	0.967	0.828**	0.978
	(0.085)	(0.058)	(0.074)	(0.057)
After the shock * S	1.060	1.003	1.091	1.006
	(0.074)	(0.046)	(0.069)	(0.042)
After the shock * T_2	1.247***	1.094**	1.224***	1.081*
	(0.074)	(0.047)	(0.067)	(0.047)
Controls	Birth-specific variables			
Observations	479	671	2409	2601

Data : PSF. Dep. var. : duration between births i and $(i+1)$. In columns 1 and 3, I consider all intervals before the shock, and the very next interval after the shock. In columns 2 and 4, I consider all intervals. Durations between marriage and first birth are always excluded (when I include them, estimates are less significant).

Sample : women below 45 years old, for whom the complete birth history is known, having at least one child from current union ; only first wives in the first two columns, monogamous and first wives in the last two columns.

"After the shock" is a time-varying variable indicating if the second wife has arrived. $S =$ (first wife age at shock - first wife age at marriage) ; $T_1 =$ min (45-first wife's age at marriage ; 60-husband's age at first marriage) ; $T_2 =$ min (45-second wife's age at marriage ; 60-husband's age at second marriage).

Controls : A dummy for each i, mother's age and age squared at birth i.

Cox estimation ; Stratified likelihood with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations.

Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 12: Testing the predictions on the birth spacing of second wives

Hazard ratios	Using S	Using n_{ini}
T_2	0.961	0.874*
	(0.032)	(0.063)
$T_1 - S$	1.011	1.166***
	(0.015)	(0.063)
S	1.043***	
	(0.015)	
n_{ini}		1.258**
		(0.119)
Specific controls	Predictors of preferences	
Additional controls	Yes	
Observations	446	213

Data : PSF. Dep. var. : duration between births i and $(i+1)$. Birth 0 corresponds to the date of marriage. Sample : second wives, below 45 years old, for whom the complete birth history is known, having at least one child from current union. In column 2, I focus on unions in which the complete birth history of the first wife is known to be able to compute n_{ini}, the first wife's number of children as the second wife arrived.
T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; S = (first wife age at shock - first wife at marriage) ; $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S.
Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth i, a dummy for each i.
Cox estimation ; Breslow method to handle ties among non-censored durations.
Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 13: Testing strategic overshooting on the number of children

	Linear	Reference $T_h - T_1 = 0$
n_w^{id}	0.195**	0.229**
	(0.095)	(0.092)
n_h^{id}	0.119**	0.145***
	(0.059)	(0.052)
T_1	0.281***	0.612***
	(0.054)	(0.114)
$T_h - T_1$	0.131**	
	(0.056)	
$T_h - T_1$ below median		1.585**
		(0.710)
$T_h - T_1$ above median		2.783**
		(1.073)
Observations	109	109
Controls	No	Yes
pval test below=above		0.141

Data : DHS. Sample : women in a monogamous union, over 40, in first union, at least one child.
Dep. Var. : total number of births. Weights. Significance levels : * p<0.10, ** p<0.05, *** p<0.01.
T_1= min (45-first wife's age at marriage ; 60-husband's age at marriage).
$T_h - T_1$ = (60 - husband's age at marriage - T_1). The median is 7 years.
Controls : husband's and wife's age at marriage.

Table 14: Testing strategic overshooting on birth spacing

	$i \geq 0$	$i > 0$	By rank of birth
n_w^{id}	1.020	1.023*	1.024*
	(0.013)	(0.013)	(0.013)
n_h^{id}	1.030***	1.031***	1.030***
	(0.005)	(0.005)	(0.005)
T_1	1.000	0.994	0.996
	(0.019)	(0.017)	(0.017)
$T_h - T_1$	1.007	1.009	
	(0.007)	(0.007)	
$(T_h - T_1) \times \{i = 1, 2\}$			1.003
			(0.008)
$(T_h - T_1) \times \{i = 3, 4\}$			1.010
			(0.011)
$(T_h - T_1) \times \{i \geq 5\}$			1.028**
			(0.014)
Observations	3424	2768	2768
Controls	Yes	Yes	Yes

Data : DHS. Sample : women in a monogamous union, first union, at least one child. Weights.
Dep. var. : duration between births i and $(i+1)$. Birth 0 corresponds to the date of marriage. In column 2, I exclude durations between marriage and first birth.
T_1= min (45-first wife's age at marriage ; 60-husband's age at marriage).
$T_h - T_1$ = 60 - husband's age at marriage - T_1.
Controls : first wife's age at marriage and a dummy for each i.
Cox estimation ; Breslow method to handle ties among non-censored durations. Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 15: Testing changes in son preference of first wives

Hazard ratios	Exactly two competitors
After the shock	0.835
	(0.175)
No son	1.399***
	(0.167)
After the shock * No son	2.074*
	(0.804)
Controls	Birth-specific variables
Observations	2444

Data : PSF. Dep. var. : duration between births i and $(i+1)$ with $i \geq 1$.
Sample : monogamous and first wives below 45 years old, for whom the complete birth history is known, having at least one child from current union ; I exclude first wives with an infertile co-wife and with more than one fertile co-wife, restricting the analysis to unions with exactly two competitors.
"After the shock" is a time-varying variable indicating if the second wife has arrived. "No son" is a dummy equal to one if the woman had no son among her first i births.
Controls : A dummy for each i, mother's age and age squared at birth i.
Cox estimation ; Stratified likelihood with baseline hazards specific to each woman ; Breslow method to handle ties among non-censored durations.
Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Table 16: Testing if second wives react to the gender composition of first wives' children

Hazard ratios	Exactly two competitors
$Boys_{ini}$	2.266***
	(0.406)
$Girls_{ini}$	1.430
	(0.452)
Specific controls	$T_2, (T_1 - S)$ and predictors of preferences
Additional controls	Yes
Observations	151

Data : PSF. Dep. var. : duration between births i and $(i+1)$. Birth 0 corresponds to the date of marriage. Sample : second wives, below 45 years old, for whom the complete birth history is known, having at least one child from current union. I focus on bigamous unions in which the complete birth history of the first wife is known to be able to observe the gender composition of the first wife's children at the time of the shock. $Boys_{ini}$ and $Girls_{ini}$ measure the number of boys and girls born to the first wife as the second wife arrived. T_2 = min (45-second wife's age at marriage ; 60-husband's age at second marriage) ; $(T_1 - S)$ = min (45-first wife's age at marriage ; 60-husband's age at first marriage)-S. Predictors of preferences : religion (husband and wife), ethnic group (husband and wife), education (husband and wife), rural dummy (husband and wife), income (husband), employment status (wife), birth cohort (husband and wife), region of residence, age at marriage (husband and wife), be in first marriage (husband and wife). Additional controls : co-residence status, work in public sector (husband), at least one child from previous union (husband and wife), having at least one dead child from current union, mother's age and age squared at birth i, a dummy for each i.
Cox estimation ; Breslow method to handle ties among non-censored durations.
Robust standard errors of the coefficients are in parentheses (clustered at the woman level). Significance levels (for hazard ratio = 1) : * p<0.10, ** p<0.05, *** p<0.01.

Appendix : For online publication

Appendix A : Additional descriptive statistics

TABLE A.1: Balancing tests between couples declaring a numerical vs. non-numerical ideal family size

Sample	Numerical ideal family size	Non-numerical ideal family size	p-value
Rural	0.540	0.658	0.000
Wealth index	3.010	2.756	0.000
Monogamous union	0.708	0.622	0.000
Head or head's spouse	0.622	0.675	0.001
No education (husband)	0.538	0.699	0.000
No education (wife)	0.637	0.791	0.000
Employed (husband)	0.884	0.925	0.000
Employed (wife)	0.409	0.408	0.969
Age (husband)	41.055	43.061	0.000
Age (wife)	30.451	31.664	0.000
Age at first marriage (husband)	27.105	26.354	0.000
Age at first marriage (wife)	18.481	17.450	0.000
Being in first marriage (husband)	0.655	0.566	0.000
Being in first marriage (wife)	0.858	0.878	0.102
Christian (husband)	0.046	0.018	0.000
Christian (wife)	0.054	0.022	0.000
Observations	1824	1484	

Data : DHS, waves 2005 and 2010. Weights.

The first column present descriptive statistics of couples in which both spouses report her ideal number of children. The second column present the same statistics for couples in which at least one spouse gave a non-numerical answer to the question "How many children would you like to have, or would you have liked to have, in your whole life ?" The third column reports the p-value of the t-tests comparing the means in both sub-samples : a low p-value indicates that they are statistically different with respect to the corresponding covariate.

Table A.2: Predictors of the ideal number of children

Sample	Husbands	Wives
Christian	-1.731***	-0.935***
	(0.481)	(0.209)
Other religion	-0.437	0.351
	(1.311)	(0.650)
Serere	-0.259	0.434**
	(0.402)	(0.198)
Poular	-0.891**	0.224*
	(0.352)	(0.131)
Mandingue	-0.337	-0.000
	(0.417)	(0.240)
Sarakole	-1.908*	0.290
	(1.024)	(0.509)
Diola	-0.728	-0.146
	(0.550)	(0.235)
Other ethnic group	-0.356	0.138
	(0.512)	(0.202)
No education	0.821***	0.421***
	(0.295)	(0.119)
Rural	0.937***	0.187
	(0.341)	(0.138)
Wealth index	-0.555***	-0.246***
	(0.138)	(0.052)
Employed	-0.265	0.158
	(0.419)	(0.110)
Head or head's spouse	0.295	-0.187
	(0.289)	(0.114)
Age at first marriage	-0.073***	-0.041***
	(0.022)	(0.014)
Being in first marriage	-0.727**	-0.047
	(0.352)	(0.146)
Monogamous union	-2.709***	-0.123
	(0.479)	(0.117)
Constant	14.739***	7.055***
	(1.286)	(0.419)
Cohort Fixed Effect	Yes	Yes
Region Fixed Effect	Yes	Yes
Observations	1928	2523
R2	0.26	0.15

Data : DHS, waves 2005 and 2010. Weights.
Dep. var : ideal number of children. Sample : respondents who gave a numerical answer to the question "How many children would you like to have, or would you have liked to have, in your whole life ?"
Reference categories are "Muslims" for the religion and "Wolof" for the ethnic group.
Significance levels : * $p<0.10$, ** $p<0.05$, *** $p<0.01$.

Appendix B : Welfare analysis

In this section, I compare the Nash equilibrium and the outcome maximizing total welfare. I focus on two quantities of interest : the total number of children ($N = n_1 + n_2$) and the relative number of children ($\Delta = n_1 - n_2$). I consider the simple case when parameters are the same for both wives : $\theta_1^h = \theta_2^h = \theta^h$, $\theta_1^n = \theta_2^n = \theta^n$ and $\epsilon_1 = \epsilon_2 = \epsilon$.

From equation 4, I derive the quantities at equilibrium :

$$N^* = n_1^* + n_2^* = \frac{N^{id} + 2\theta^h n_h^{id} + \theta^n N^{nat}}{1 - \epsilon + 2\theta^h + \theta^n}$$

$$\Delta^* = n_1^* - n_2^* = \frac{\Delta^{id} + \theta^n \Delta^{nat}}{1 + \epsilon + \theta^n}$$

Where $N^{id} = n_1^{id} + n_2^{id}$, $N^{nat} = n_1^{nat} + n_2^{nat}$, $\Delta^{id} = n_1^{id} - n_2^{id}$ and $\Delta^{nat} = n_1^{nat} - n_2^{nat}$.

I further define the total welfare function as $W(n_1, n_2) = u_1(n_1, n_2) + u_2(n_2, n_1)$, where $u_i(n_i, n_{-i})$ is the utility of wife i. By maximizing $W(n_1, n_2)$ over n_1 and n_2, I find :

$$N^{Opt} = n_1^{Opt} + n_2^{Opt} = \frac{(1-\epsilon)N^{id} + 4\theta^h n_h^{id} + \theta^n N^{nat}}{(1-\epsilon)^2 + 4\theta^h + \theta^n}$$

$$\Delta^{Opt} = n_1^{Opt} - n_2^{Opt} = \frac{(1+\epsilon)\Delta^{id} + \theta^n \Delta^{nat}}{(1+\epsilon)^2 + \theta^n}$$

The same elements drive N and Δ when I maximize total welfare and when I compute the Nash equilibrium. But the weights given to each element differ. The table below summarizes the drivers and their weights in both cases.

Weight on each driver	Nash equilibrium	Welfare-maximizing outcome
Drivers of N		
$N^{id}/(1-\epsilon)$	$\frac{(1-\epsilon)}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{(1-\epsilon)^2}{(1-\epsilon)^2+4\theta^h+\theta^n}$
n_h^{id}	$\frac{2\theta^h}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{4\theta^h}{(1-\epsilon)^2+4\theta^h+\theta^n}$
N^{nat}	$\frac{\theta^n}{(1-\epsilon)+2\theta^h+\theta^n}$	$\frac{\theta^n}{(1-\epsilon)^2+4\theta^h+\theta^n}$
Drivers of Δ		
$\Delta^{id}/(1+\epsilon)$	$\frac{(1+\epsilon)}{(1+\epsilon)+\theta^n}$	$\frac{(1+\epsilon)^2}{(1+\epsilon)^2+\theta^n}$
Δ^{nat}	$\frac{\theta^n}{(1+\epsilon)+\theta^n}$	$\frac{\theta^n}{(1+\epsilon)^2+\theta^n}$

Compared to the welfare-maximizing outcome, the total number of children at equilibrium depends too much on the preferences of the wives, $N^{id}/(1-\epsilon)$, and too little on the preferences of the husband, n_h^{id}. The relative number of children depends too much on the difference in wives' natural fertility, Δ^{nat}, and too little on the difference in wives' preferences, Δ^{id}.

To sum up, if wives were to agree on maximizing total welfare instead of maximizing their own utility, the total number of children would be closer to the husband's preferences. The relative number of children would reflect more the difference in wives' preferences and less the difference in natural fertility.

Are there too many or too few children at equilibrium ? We have :

$$N^* - N^{Opt} = \frac{(\frac{N^{id}}{1-\epsilon} - n_h^{id}).2\theta^h(1-\epsilon^2) + (\frac{N^{id}}{1-\epsilon} - N^{nat}).\theta^N\epsilon(1-\epsilon) + (N^{nat} - n_h^{id}).2\theta^h\theta^n}{((1-\epsilon)^2 + 4\theta^h + \theta^n)(1-\epsilon+2\theta^h+\theta^n)}$$

The comparison between N^* and N^{Opt} depends on the relative values of $\frac{N^{id}}{1-\epsilon}$, n_h^{id} and N^{nat}. In particular, if $\frac{N^{id}}{1-\epsilon} \geq N^{nat} \geq n_h^{id}$, then $N^* \geq N^{Opt}$; whereas if $\frac{N^{id}}{1-\epsilon} \leq N^{nat} \leq n_h^{id}$, then $N^* \leq N^{Opt}$. The first statement is all the more likely to hold as ϵ rises. In the empirical tests, I found that ϵ is large enough to induce a positive strategic reaction. One plausible value for

ϵ is therefore its upper bound. When $\epsilon = 1$, we have :

$$N^{*} - N^{Opt} = \frac{N^{id}.(4\theta^{h} + \theta_{n}) + (N^{nat} - n_{h}^{id}).2\theta^{h}\theta^{n}}{(2\theta^{h} + \theta^{n})(4\theta^{h} + \theta^{n})}$$

Using the values of the parameters estimated on monogamous unions ($\theta^{h} \approx 1/2$ and $\theta^{n} \approx 3$), it means that $N^{*} \geq N^{Opt}$ iif $N^{id} \geq \frac{3}{5}(n_{h}^{id} - N^{nat})$. This condition is likely to hold in the vast majority of cases given that preferences in polygamous unions are on average 5 or 6 children for each wife, and 12 children for the husband. To sum up, a deficit of children would be observed at equilibrium only when n_{h}^{id} reaches uncommonly high values. In general, the non-cooperative model leads to a surplus of children with respect to the outcome maximizing total welfare.