Intergenerational Mobility and Urban Segregation*

Emeline BEZIN† and Fabien MOIZEAU‡

December 2014
Second Draft

Abstract

We study the relationship between intergenerational mobility and urban segregation. To this aim, we develop a model of neighbourhood formation and preference transmission. The key feature is that incentives the parents face to transmit their trait to their offspring depend on the endogenous social composition on the neighbourhood. The urban equilibrium that emerges at each date is segregated. It turns out that some urban areas are characterized by better social mobility prospects than others. We show that the joint dynamics of segregation and distribution of traits in the whole population exhibits multiple history-dependent steady-states. Finally, segregation is not always efficient. This appeals to public policies that would give rise to socio-economic compositions of neighbourhoods more favorable to social mobility, as for instance group-based policies.

---

*We are very grateful to David de la Croix, Luisa Gagliardi for their helpful comments. A former version of the paper entitled "Urban Segregation and Cultural Dynamics" was presented at the "Social Interactions and Urban Segregation" summer school (Rennes, 2014). We thank participants at this conference. We also thank participants at the 9th Meeting of the Urban Economics Association (Washington, 2014) and seminar participants at ENS Cachan, BETA (Strasbourg). Financial support from the Agence Nationale de la Recherche (ANR-12-INEG-0002) is gratefully acknowledged.

†IRES, Université Catholique de Louvain and SMART-INRA, email: emeline.bezin@rennes.inra.fr
‡CREM (Condorcet Center), Université de Rennes 1 and Institut Universitaire de France, email: fabien.moizeau@univ-rennes1.fr.
1 Introduction

Recent studies on intergenerational mobility in the US have emphasized that the place of residence matters for the individuals’ prospects of social mobility (see Sharkey, 2008, or Chetty, Hendren, Kline and Saez, 2014). In particular, Chetty, et al. (2014) using data from the federal income tax records over the period 1996-2012, emphasize the great spatial variation of social mobility patterns. They also show that high mobility areas have less residential segregation, less income inequality, better primary schools, greater social capital and greater family stability. These results suggest that the social composition of neighbourhood and the family background play a major role in the intergenerational transmission of economic status.

Furthermore, some studies have emphasized the interdependency between the quality of the neighbourhood and the parental involvement into education of their offspring. Using the UK National Child Development Study, Patacchini and Zenou (2011) find that parents invest more in the education of their child when residing in a high quality neighbourhood. Ioannides and Zanella (2008) study determinants of location choices using the PSID data. Their findings suggest that parents with children search for neighbourhoods whose attributes are favourable to the production of human capital production and the transmission of parents' cultural traits.

In this paper, we study the relationship between intergenerational mobility and the spatial socio-economic segregation. To this end, we develop a theoretical set up that formalizes the interdependency between quality of the neighbourhood and parental involvement in the intergenerational transmission of education.

We consider overlapping generations of individuals who live two periods. When child, an individual decides which level of educational effort to exert. Two key factors influence incentives to exert educational effort, i.e. the taste for education and peer effects. Children differ with respect to their taste for education. Either, children have a taste for education and benefit from an extra gain when they exert high effort or children dislike education and enjoy an extra gain when they exert no effort. This heterogeneity of preferences for education can be justified in different ways. First, there is empirical evidence supporting gender differences in preferences for education. For instance, Zafar (2012) find that heterogeneity of preferences between men and women account for the gender gap in choice of college major among students at Northwestern University. Furthermore, some studies reveal that cultural variables are responsible for differences in gender educational achievement in developing countries (Dollar and Gatti, 1999, Norton and Tomal, 2009, and Cooray and Potrafke,
Second, the literature on racial inequality emphasizes that ethnic minorities impose costs on members who adopt majority behaviours (see Fryer and Torelli, 2010, for an empirical analysis of ‘Acting White’, and Battu and Zenou, 2010, and Battu et al. 2007 for empirical evidence on oppositional identities). Third, heterogeneity of lifestyle preferences (importance given to leisure, housework) could explain why individuals do not attach the same value to education.

These preferences are transmitted following the cultural transmission mechanism à la Bisin-Verdier (2001)\(^1\). A child acquires some cultural trait either from her/his parent’s socialization investment called direct vertical socialization or from encountering role models in the neighbourhood called the oblique socialization. There are also peer effects generated by the neighbourhood which impact the cost of education. A key feature is that peer effects in education vary spatially and thus incentives to exert educational effort differ across space. When parent, an individual decides how much to invest into socializing her/his offspring to her/his own taste for education. The incentive to transmit ones’ own preferences is due to imperfect altruism. Parents are able to correctly assess the optimal choices of their child, but only through the filter of their own preferences. The incentives driving the choice of socialization investment depend on the endogenous socio-economic composition in two ways. On the one hand, peer effects influence the child’s future well-being which is taken into consideration by parents. Peer effects make the parents with a taste, respectively a distaste, for education more, respectively less, willing to exert a socialization choice in order to transmit their trait. On the other hand, there is an opposite effect generated by the oblique transmission mechanism. The more numerous the population with the same cultural trait in the neighbourhood the lower the incentives to transmit her/his trait for a parent. This is called the cultural substitution effect. The decision to exert the socialization effort depends on the trade-off between peer effects and the cultural substitution effect. When peer effects dominate, the incentive for parents to make a socialization effort increases with the fraction of agents with similar preferences. This gives rise to cultural complementarity. When the cultural substitution effect dominates, the incentive to make a socialization effort is lower when the share of agents with the same preferences increases. This creates cultural substitutability.

Furthermore, socio-economic composition of each neighbourhood is endogenous as parents also choose the place of residence of the family in order to get the best combination (from their point of view) of peer effects and cultural substitution effect. It turns out that location decisions depend

\(^1\)Note that if racial characteristics or genders are transmitted genetically, ethnic traits and gender norms we mentioned before are rather transmitted through social interactions.
on both segregation and integration forces. In addition to peer effects which generate segregation (see for instance de Bartolome, 1990, or Bénabou, 1996), our model highlights an integration and a segregation force. First, due to imperfect empathy, parents prefer their child to acquire their trait rather than the other one. This leads them to find profitable to live in an area inhabited by parents with similar preferences as it would reinforce the probability to transmit their trait by oblique transmission. This first effect, the oblique transmission effect, is a force toward segregation. Second, due to cultural substitution, incentives to directly transmit ones’ own trait are lower when the offspring is more likely to pick the same trait in the neighbourhood population. In such a case, the neighbourhood choice and the socialization effort are substitutable instruments of vertical transmission. Therefore parents may prefer to live in a mixed urban area where, even though oblique transmission is not so effective, they can still transmit their own preferences with a high probability by exerting a high socialization effort. This is a force toward integration.

Our results are threefold. First, we characterize the urban equilibrium that emerges at each date. Assuming that segregation forces outweigh the integration one, people sort themselves into unequal urban areas. The implied equilibrium is thus segregated. We show that the patterns of socialization choices in urban areas depend on the fraction individuals with a taste for education in the whole population. For both low and high fractions of such individuals, segregation does not provide incentives to transmit the trait for education as either peer effects are too low or cultural substitution is too high. In such a case, location decision and socialization choice are substitutable instruments of direct vertical socialization. While for intermediate values of the fraction of individuals with a taste for education in the whole population, the concentration of these individuals in the same urban area can generate peer effects high enough to exert socialization effort. In this case, location decision and socialization effort are complementary instruments of direct vertical socialization. For any segregated equilibrium, individuals with a taste for education are more likely to reside in an urban area with a higher education rate. Intergenerational mobility also differs among urban areas. A child living in places where the education rate is higher is more likely to acquire the education trait whatever her/his parent’s preferences.

Second, we study the joint dynamics of segregation and distribution of preferences for education in the whole population. Given the size of the population with a taste for education at date $t$, there is a particular combination of peer effects and cultural substitution effect in each urban area leading to various socialization decisions. Hence, a segregated equilibrium will drive a particular population dynamics giving rise to a new distribution of preferences at date $t + 1$. In turn, the segregated
equilibrium that emerges at date $t + 1$ is characterized by a new pattern of socialization choices in urban areas. We show that the dynamics exhibits multiple history-dependent steady states. For low fractions of people with a taste for education at date 0, peer effects are too low to provide incentives to exert a socialization effort whatever the place of residence. The initial distribution of traits remains overtime and the population is trapped in a low level of education. We call these cities ‘stationary mobile’ cities reflecting the fact that the number of children experiencing downward social mobility exactly equals those experiencing upward social mobility. For intermediate initial values of the fraction of people with a taste for education, the segregated city can be such that peer effects exceed the cultural substitution effect and provide incentives to exert a high socialization effort. The population with a taste for education thus expands. The number of people experiencing upward social mobility se cities are called ‘dynamically mobile’ cities. Nonetheless, this expansion may be stopped once a threshold fraction of people with a taste for education has been reached. Indeed, for higher fractions of agents with a taste for education, the cultural substitution effect outweighs peer effects depriving every urban areas from incentives to exert socialization choices. Above this threshold, the distribution of traits is stationary and the city is ‘stationary mobile’. Third, we address the issue of efficiency. The efficiency criterion we use is long run education. We show that laissez-faire which leads people to segregate is not always optimal. The reason is that it may lead to too much concentration of agents with a taste for education which lowers incentives to exert a transmission effort due to a strong cultural substitution effect. There is thus room for policies which would make the socio-economic composition of urban areas provide the incentives for the transmission of the taste for education.

Our paper belongs to two strands of literature. It is related to the literature on neighbourhood effects and endogenous socio-economic segregation which explains how local interactions drive spatial segregation and persistent inequality (see for instance, Loury, 1977, Bénabou, 1993, 1996a,b, Durlauf, 1996). We depart from these works as our set-up allows to analyse how intergenerational transmission mobility is influenced by spatial segregation. Our paper is also related to the literature on cultural transmission which has been impulsed by Bisin and Verdier (2001). Bisin, Patacchini, Verdier, and Zénou (2011) study the impact of segregation on the persistence of oppositional values. However the socio-economic segregation is taken as exogenous. Our set-up endogenizes the socio-economic composition of the neighbourhood that drives the process of preferences transmission.

The plan of the paper is as follows. In the following section we set up the model. In section 3, we characterize the urban equilibrium which emerges at each date $t$. Then in section 4, we study
the dynamics of urban segregation and distribution of traits. In section 5, we address the issue of
efficiency of the urban equilibrium. Section 6 concludes.

2 The Setup

2.1 The City

The city is comprised of two residential areas indexed by $j = 1, 2$. We consider that landowners are
absent and without loss of generality we normalize the opportunity cost of building a house to 0.
Houses are identical across the city. The inelastic supply of houses within a residential area is of
mass $L$. This land-market is a closed-city model where the population of the city is a continuum
of families of mass $M$. Each family, comprised of a parent and a child, lives in one and only one
house. The city can accommodate the entire population and we assume for the sake of simplicity
that $L = M/2$. Agents live two periods. When a child, an individual faces an educational discrete
choice. When an adult, an individual has to decide in which neighbourhood her family resides and
the effort he will exert to transmit her cultural trait.

2.2 Children’s Educational Choice

Preferences. Children have to decide whether they exert a high educational effort denoted by $\bar{e}$
or a low effort $e < \bar{e}$ depending on their preferences for education. Children of type $a$, who enjoy
education, derive an extra benefit from exerting a high effort which is equal to $a$. By contrast,
children of type $b$, dislike education and derive a benefit $b$ from exerting a low education effort.\(^2\) Let
$U_{ij}^a$ denote preferences of a child with trait $i = \{a, b\}$ living in area $j = 1, 2$ at date $t$. Preferences
are defined as follows:

\[
U_{ij}^a = p(e)w_r + (1 - p(e))w_p + ap(e) - (1 - \lambda_i^1)c_{1_{e=\bar{e}}}
\]

and

\[
U_{ij}^b = p(e)w_r + (1 - p(e))w_p + (1 - p(e))b - (1 - \lambda_i^1)c_{1_{e=\bar{e}}}
\]

While exerting effort $e$, a child is rich and earns income $w_r$ with probability $p(e)$. Otherwise, he
is poor with income $w_p < w_r$. We assume for the sake of simplicity that $p(\bar{e}) = 1$ and $p(e) = 0.$

\(^2\)Let us mention that it is crucial to have individuals who value differently education. This implies that incentives
to exert an educational effort are not the same among children. Our results would remain under the assumption that
costs of education differ among type $a$ individuals and type $b$ individuals.
Exerting effort $\bar{e}$ incurs the cost of education $(1 - \lambda^j_t)c$, with $c \in \mathbb{R}_+$ and $\lambda^j_t$ the fraction of children exerting effort $\bar{e}$ in neighbourhood $j$. We thus allow for peer effects in education by assuming that the cost reduces with the fraction of educated children in the neighbourhood, $\lambda^j_t$. We assume that exerting effort $e$ incurs no cost. Finally, this cost function is the same for agents of type $a$ and $b$.

We denote by $N_t$ (resp. $N^j_t$) the number of agents with trait $a$ in the whole population (resp. in area $j$) at time $t$ and by $Q_t$ (resp. $q^j_t$) the fraction $N_t/L$ ($N^j_t/L$) of these agents at time $t$.

**Educational Choice.** At date $t$, a child with trait $a$ residing in area $j$ decides to educate if and only if

\[ w_r - (1 - \lambda^j_t)c + a > w_p \]

while a child with trait $b$ residing in area $j$ exerts high effort if and only if

\[ w_r - (1 - \lambda^j_t)c > w_p + b. \]

We make the following assumptions

**Assumption 1** For any $w_r$ and $w_p$,

(i) $a$ is such that $w_r - c + a > w_p$,

(ii) $b$ is such that $w_r < w_p + b$.

This implies that, for any $\lambda^j_t \in [0, 1]$, any type $a$ individual exerts a high effort whereas any type $b$ agent exerts a low effort. One can deduce the fraction of children who educate is

\[ \lambda^j_t = q^j_t. \]

Assumption 1 is not crucial for our results but allows us to get straightforwardly the rate of

---

3In the literature, one usually assumes that the derivative of the cost with respect to $\lambda_j$ is higher for trait $a$ individuals than for trait $b$ individuals in order to obtain segregated equilibria (see Bénabou, 1993). Here we choose the same cost function so as to emphasize new mechanisms toward segregation.
education. The only restriction we impose is that there is no corner solution such that the whole young population chooses high effort. We could have assumed that children differ with respect to innate ability and only the smarter individuals educate (with a higher fraction of type a young individuals who decide to educate). This would not change our results.

2.3 Parents’ transmission decision and location choice

At any date \(t\), parents who differ with respect to both income and preferences, make two decisions. They choose the location \(j\) where they pay the land rent \(\rho_j^t\) and decide to make a socialization effort.

**Dynamics of Preferences.** The transmission of preferences follows the line of the model introduced by Bisin and Verdier (2001). Intergenerational transmission of trait \(i\) is the result of social interactions which arise at two levels. The child is first exposed to vertical socialization by his/her parent. The probability that the latter directly transmits her/his trait is \(d^i\). If not socialized within the family (with a probability \(1 - d^i\)), the child is obliquely socialized picking the trait of a role model chosen randomly in the neighbourhood \(j\). The probability to be obliquely socialized to trait \(a\) (resp. \(b\)) in neighbourhood \(j\) is \(q_j\) (resp \(1 - q_j\)) the fraction of agents with trait \(a\) (resp. \(b\)) in this neighbourhood. The transition probabilities are given by

\[
P_{j\text{at}}^i = d^a + (1 - d^a)q_j^i \quad \text{and} \quad P_{abt}^i = (1 - d^a)(1 - q_j^i)
\]

(1)

\[
P_{bbt}^i = d^b + (1 - d^b)(1 - q_j^i) \quad \text{and} \quad P_{bat}^i = (1 - d^b)q_j^i.
\]

(2)

In particular, \(P_{aat}^i\) denotes the probability that a child from a type \(a\) family is socialized to type \(a\) at time \(t\).

In this model, direct transmission is the result of a choice. More specifically, parents exert a discrete effort \(d^a \in \{0, \tau\}\) in order to transmit their preferences. The incentive to transmit ones’ own preferences comes from imperfect empathy which means that parents are able to correctly assess the optimal choices of their child but through the lens of their own preferences. Let us denote by \(V_{i'i}^{j,t+1}\) the gain for a parent of type \(i\) to have at date \(t + 1\) a child of type \(i'\) in neighborhood \(j\). Moreover, we assume that parents are myopic, namely \(V_{i'i}^{j,t} = V_{i'i}^{j,t+1}\), \(\forall i \in \{a, b\}\).

\(^4\)The latter assumption allows us to ignore self-fulfilling expectations equilibria. This would complicate the analysis.
Preferences. We denote by $U_{i;z}^{i,z}(\rho_i^j, d^i)$, the utility at date $t$ of a parent with trait $i$ and income $w_z$, $z = r, p$, who lives in neighbourhood $j$ and exert the socialization effort $d^i$. Omitting time index for transition probabilities and gains $\nu_{ii}^j$, we have for a trait $a$ parent

$$U_{i;z}^{a,z}(\rho_i^j, d_a) = w_z - \rho_i^j + P_{aa}^j V_{aa}^j + P_{ab}^j V_{ab}^j - \Theta(d^a),$$

(3)

where $\Theta(.)$ is the socialization cost with $\Theta(0) = 0 < \theta = \Theta(\tau)$. Socialization gains are given by

$$V_{aa}^j = w_r - (1 - \lambda_i^j)c + a$$

$$V_{ab}^j = w_p.$$

Note that the fact that parents are myopic implies that they consider that peer effects influencing their child’s educational effort is the rate of education in their own generation, $\lambda_i^j$.5

Let us also stress that letting $\Delta V_i^j \equiv V_{aa}^j - V_{ab}^j$ and $\Delta w \equiv w_r - w_p$, and given that $\lambda_i^j = q_i^j$, we have

$$\Delta V_i^j = w_r - w_p - (1 - q_i^j)c + a.$$ (4)

We thus see that imperfect empathy which amounts to have $\Delta V_i^j > 0$ is guaranteed by Assumption 1.

A parent with trait $b$ and income $w_z$ who lives in neighbourhood $j \in \{1, 2\}$ has utility

$$U_{i;z}^{b,z}(\rho_i^j, d_b) = w_z - \rho_i^j + P_{bb}^j V_{bb}^j + P_{ba}^j V_{ba}^j - \Theta(d^b),$$

(5)

with

$$V_{bb}^j = w_p + b$$

$$V_{ba}^j = w_r - (1 - \lambda_i^j)c.$$

without introducing any key feature into the model.

5In any cases, the child’s effort is a positive function of the rate of the education $\lambda_i^j$. 

9
Letting $\Delta V^j_b \equiv V^j_{bb} - V^j_{ba}$, we have

$$\Delta V^j_b(q^j_t) = -(\Delta w - (1 - q^j_t)c - b)$$

(6)

Given Assumption 1, we also satisfy imperfect empathy for parents of type $b$, i.e. $\Delta V^j_b > 0$.

**Socialization choice of type a parents.** At date $t$, a parent with type $a$ exerts an effort in neighborhood $j$ if and only if

$$U^a_{t}(a_j^t, \tau) \geq U^a_{t}(a_j^t, 0).$$

Given (1) and (3), the above inequality writes

$$(1 - q^j_t)\tau \Delta V^j_a - \theta \geq 0.$$

Given (4), the above inequality can be expressed as follows

$$(1 - q^j_t)\tau (\Delta w - (1 - q^j_t)c + a) - \theta \geq 0,$$

(7)

The choice of the socialization effort involves a trade-off between the gain from socialization, denoted by $(1 - q^j_t)\tau (\Delta w - (1 - q^j_t)c + a)$ and the cost of transmission $\theta$. In particular, the gain from socialization depends on the fraction of agents $a$ through two opposite effects. The negative effect is the cultural substitution effect captured by the term $(1 - q^j_t)$. As the fraction of agents with the same trait increases, socialization by peers become more effective so that the incentive to make a high socialization effort is reduced. The positive effect is the peer effects captured in $V^j_a$ by $-(1 - q^j_t)c$.

All else equal, a child with preferences for education is better-off whenever the fraction of educated people in the neighbourhood increases. Hence, peer effects give rise to to strategic complementarity, i.e. $\partial \Delta V^j_a / \partial q^j_t > 0$.

We assume the following

**Assumption 2** Parameters $c, w_r, w_p, \tau, \theta$ are such that

(i) Equation (7) admits two positive roots $\tilde{q}_1$ and $\tilde{q}_2$.

(ii) $\tilde{q}_1 < 1/2 < \tilde{q}_2 < 1, \tilde{q}_1 + \tilde{q}_2 < 1$. 


The Appendix provides a more detailed analysis of (7). Under Assumption 2, we consider intermediate values of peer effects relative to the cost of socialization. In particular, when peer effects are relatively low, Assumption 2 implies that there is no incentive to socialize children to trait $a$. However, when peer effects are higher and overweight the cultural substitution effect, parents of type $a$ will have an incentive to make a positive socialization effort.

We deduce the optimal socialization choices of parents of type $a$ in neighbourhood $j$

$$d^a = \begin{cases} 
0 \iff q^j < \tilde{q}_1 \text{ or } q^j > \tilde{q}_2, \\
\tau \iff q^j \in [\tilde{q}_1, \tilde{q}_2]. 
\end{cases} \quad (8)$$

The optimal effort function of agents of type $a$ is a non-monotonous step function of $q^j$, the fraction of these agents in the neighbourhood. For low values of $q^j$, the peer effects dominate. In such a case, the incentives to exert an effort is an increasing function of $q^j$. Hence the optimal effort is positive if and only if $q^j$ is sufficiently high, i.e. $q^j > \tilde{q}_1$ (peer effects are high). There is cultural complementarity. For higher values of $q^j$, the cultural substitution effect becomes stronger and can overweight peer effects. The incentives to make an effort then becomes a decreasing function of $q^j$. Therefore, the optimal effort is still positive if and only if $q^j$ is sufficiently low, i.e. $q^j < \tilde{q}_2$ (the cultural substitution effect is low) and becomes nil when $q^j > \tilde{q}_2$ (the cultural substitution effect is strong). There is cultural substitutability$^6$.

**Socialization choice of type $b$ parents.** At time $t$, a parent with type $b$ and income $w_z$ exerts an effort in neighbourhood $j$ if and only if

$$U^{b,z}_t(\lambda^j_t, \tau) \geq U^{b,z}_t(\rho^j_t, 0)$$

which is equivalent to

$$q^j_t \tau \Delta V^j_b(\lambda^j_t) - \theta \geq 0.$$ 

Given (6), we obtain

$$-q^j_t \tau (\Delta w - (1 - q^j_t) c - b) - \theta \geq 0. \quad (9)$$

Gains from socialization are now expressed by $-q^j_t \tau (\Delta w - (1 - q^j_t) c - b)$. They depend on a cultural

$^6$The empirical literature has provided evidence of such a non-monotonic socialization effort. In particular, Bisin, Topa and Verdier (2004), find an inverted-U relationship between socialization rates to transmit some religious trait and the share of this religious trait in the US population.
substitution effect captured by $q^j_t$ (that is, parents with trait $b$ face fewer incentives to socialize their child when there are more type $b$ individuals in the neighbourhood population, i.e. when $q^j_t$ is higher) and peer effects captured in $\Delta V^j_b$ by $-(1-q^j_t)c$.

We assume the following

**Assumption 3** Parameters that $\tau$, $\Delta w$, $c$, $b$ and $\theta$ are such that parents with trait $b$ do not exert any socialization effort whatever the value of $q^j_t$.

We thus consider a case where education is profitable enough so that agents of type $b$ never transmit their trait.\footnote{We could consider other situations. Nevertheless, the main results are already captured by this simple case. Furthermore, the fact that some non-educated people would make an effort so that their child would be non-educated is discussed in the literature (see Patacchini and Zenou, 2011)}

The dynamics of the fraction of the population with trait $a$ in neighbourhood $j \in \{1, 2\}$ is finally given by

$$q^j_{t+1} = \begin{cases} 
q^j_t, & \text{if } q^j_t < \tilde{q}_1, \\
q^j_t + q^j_t (1 - q^j_t) \tau, & \text{if } \tilde{q}_1 \leq q_t \leq \tilde{q}_2 \\
q^j_t, & \text{if } q^j_t < \tilde{q}_2. 
\end{cases} \quad (10)$$

**Parents’ Location Choice.** At any date $t$, parents choose where to reside. They must solve the following program

For type $a$ parent, \[ \max_j u(w - \rho^j_t) + P^j_{aa} V^j_{aa} + P^j_{ab} V^j_{ab} - \Theta(d^a), \quad \text{with } d^a \text{ solution of } (7). \]

For type $b$ parent, \[ \max_j u(w - \rho^j_t) + P^j_{bb} V^j_{bb} + P^j_{ba} V^j_{ba} - \Theta(d^b), \quad \text{with } d^b = 0. \]

We have $Q_t$, the fraction of agents with trait $a$ in the whole population, such that $q^1_t + q^2_t = Q_t$. Without loss of generality, we impose that $q^1_t \geq q^2_t$ and $q^2_t = 0$. The urban equilibrium is defined as follows:

**Definition 1** At any date $t$, given $Q_t$, the urban configuration $[\rho^*_t, q^{1*}_t, d^{1,a}_t, d^{2,a}_t]$ is an equilibrium if no one wants to move and change her socialization choice.
In order to obtain the urban equilibria, we characterize the willingness to pay to live in urban area 1, denoted by $p_i$ for $i \in \{a, b\}$. It is such that a type $i$ parent is indifferent between both neighbourhoods. For a type $a$ individual, given that $P_{ab} = 1 - P_{aa}$, we obtain

$$p_{it}^a = P_{aa}^1 (\Delta V_a^1) - P_{aa}^2 (\Delta V_a^2) + V_{ab}^1 - V_{ab}^2 - (\Theta (d_{1a}) - \Theta (d_{2a}))$$

with $d_{ja}, j \in \{1, 2\}$, solution of (7). For a type $b$ individual, we get

$$p_{it}^b = P_{bb}^1 (\Delta V_b^1) - P_{bb}^2 (\Delta V_b^2) + V_{ba}^1 - V_{ba}^2.$$

Given (1), (2), (4), (6), Assumption 3, and after some manipulation the bid-rent differential can be expressed as follows

$$p_{it}^{b1} - p_{it}^{a1} = -(q_{it}^1 - q_{it}^2)(b + a) - (d_{1a}^1(1 - q_{it}^1)\Delta V_a^1 - \Theta (d_{1a}^1)) + d_{2a}^2(1 - q_{it}^2)\Delta V_a^2 - \Theta (d_{2a}^2).$$

(11)

The social composition of the neighbourhood measured by $q_{it}$ impacts the bid-rent differential via both transition probabilities and the gain from socialization. Integration as well as segregation forces are at play in the bid-rent differential (11).

First, there is an oblique transmission effect which is due to imperfect empathy. It is captured by the term $-(q_{it}^1 - q_{it}^2)(b + a)$. Given that $q_{it}^1 \geq q_{it}^2$ transmission of trait $a$ (resp. $b$) by peers is relatively higher in urban area 1 (resp. in urban area 2) for individuals with trait $a$ (resp. trait $b$). Due to cultural intolerance, parents with trait $a$ (resp. trait $b$) prefer to have a child of type $a$ (resp. $b$). Hence, they find profitable to live in area 1 (resp. 2) since it provides a higher probability to transmit their trait by oblique transmission. Therefore, the oblique transmission effect favours segregation.

Second, the bid-rent differential depends on the net gains of socialization in each urban area. They either increase or reduce $p_{it}^{b1} - p_{it}^{a1}$ depending on the magnitude of peer effects and cultural substitution effects in both urban areas. On the one hand, as $q_{it}^1 \geq q_{it}^2$, peer effects in urban area 1 are stronger than those in urban area 2. Other things being equal, incentives to transmit the trait are stronger for parents living in urban area 1. Hence, the gains from socialization are higher in urban area 1. This is a force toward segregation. On the other hand, the cultural substitution effect is stronger in urban area 1. Other things being equal, parents are then more likely to exert a high
socialization effort in urban area 2. This leads to higher net gains from socialization in urban area 2. This is a force toward integration. In what follows let us assume that the segregation forces are higher. This formally requires

**Assumption 4** Parameters $a$, $b$, $c$, $\theta$, $\tau$ and $\Delta w$ are such that

$$4(a + b)^2 > \tau(a + \Delta w) - 4c\theta$$

This amounts to say that the oblique transmission effect due to imperfect empathy is high compared to gains of socialization which increase with $\tau(a + \Delta w)$ and decrease with the cost of education $c$.

**Proposition 1** At any date $t$,

(i) For any $Q_t \in [0, \tilde{q}_1]$, $[\rho^*_t = \bar{p}^{b,1}_t, q^{1*}_t = Q_t, d^{1,1}_a = 0, d^{2,2}_a = 0]$ is the unique stable equilibrium.

(ii) For any $Q_t \in [\tilde{q}_1, \tilde{q}_2]$, $[\rho^*_t = \bar{p}^{b,1}_t, q^{1*}_t = Q_t, d^{1,1}_a = \tau, d^{2,2}_a = 0]$ is the unique stable equilibrium.

(iii) For any $Q_t \in [\tilde{q}_2, 1]$, $[\rho^*_t = \bar{p}^{b,1}_t, q^{1*}_t = Q_t, d^{1,1}_a = 0, d^{2,2}_a = 0]$ is the unique stable equilibrium.

(iv) For any $Q_t \in [1, 1+\tilde{q}_1]$, $[\rho^*_t = \bar{p}^{a,1}_t, q^{1*}_t = 1, d^{1,1}_a = 0, d^{2,2}_a = 0]$ is the unique stable equilibrium.

(v) For any $Q_t \in [1+\tilde{q}_1, 1+\tilde{q}_2]$, $[\rho^*_t = \bar{p}^{a,1}_t, q^{1*}_t = 1, d^{1,1}_a = 0, d^{2,2}_a = \tau]$ is the unique stable equilibrium.

(vi) For any $Q_t \in [1+\tilde{q}_2, 2]$, $[\rho^*_t = \bar{p}^{a,1}_t, q^{1*}_t = 1, d^{1,1}_a = 0, d^{2,2}_a = 0]$ is the unique stable equilibrium.

According to this Proposition, the segregated equilibrium is the unique stable equilibrium. However, the characteristics of the segregated equilibrium given by the social composition $q^1_t$ and the socialization choices in urban areas 1 and 2 vary, depending on the number of trait $a$ individuals in the population measured by $Q_t$.\(^{10}\)

When $Q_t \in [0, \tilde{q}_1]$, the number of trait $a$ individuals is relatively small. For any social composition such that $q^1_t \geq q^2_t$, incentives to exert a socialization effort are not sufficient in both urban areas.

---

\(^8\)Note that segregation and integration forces would be at stake in a model with a continuous socialization effort, with the only difference that their magnitude would be continuous functions of $q$. For instance, for low values of $q$, agents $a$ would exert a higher effort in area 1 due to peer effects, thus favoring segregation. If agents of type $b$ were exerting a high educational effort in area 1, a new integration force would be at play. Considering a case where the segregation forces overcome the integration ones would require similar assumptions on the parameters (notably the gain of socialization not too high).

\(^9\)Later on, we study the case where integration forces are high relatively to the segregation ones.

\(^{10}\)Importantly, as socialization choice is discrete, the bid-rent differential (11) is a step function of $q^1_t$. Hence, in order to obtain the urban equilibrium given a certain $Q_t$, we must characterize the bid-rent differential for any $q^1_t$ and $q^2_t$ such that $q^1_t + q^2_t = Q_t$. In other words, the sorting condition that provides a ranking of bid-rent slopes is no more sufficient to characterize the urban equilibrium.
The oblique transmission effect is thus the only segregative force at play. Individuals with trait $a$ find profitable to live in area 1 as it provides a higher probability to transmit their trait by oblique transmission while trait $b$ agents prefer urban area 2 where the probability is higher to transmit their trait. It turns out that at equilibrium all trait $a$ individuals locate in urban area 1 and do not exert any socialization choice. Urban area 2 is characterized by a zero education rate. Hence, for such range of $Q_t$, location decision and socialization choice are substitutable instruments of direct vertical socialization.

When $Q_t \in [\tilde{q}_1, \tilde{q}_2]$, incentives to socialize children vary with respect to the social composition arising in urban areas. Three cases must be taken into consideration to deduce the urban equilibrium. First, if all trait $a$ individuals live in urban area 1, i.e. $0 = q_t^a < q_1^a < q_2^a$, in urban area 1 peer effects are sufficiently high to compensate the cultural substitution so that type $a$ individuals exert a socialization effort. The segregation forces, namely peer effects and the oblique transmission effect outweigh the integration force, namely the cultural substitution effect. Second, the type $a$ individuals live in both urban area but the social composition is such that gains of socialization are not high enough to incite parents to exert a socialization effort, i.e. $q_t^2 \leq q_1^a < \tilde{q}_1$. The bid-rent differential (11) depends only on the oblique transmission effect. Third, we can have the case where the type $a$ individuals are distributed among urban areas and exert a socialization effort in both urban areas, i.e. $\tilde{q}_1 < q_t^2 < q_1^a < \tilde{q}_2$. The cultural substitution effect lowers the incentive to live in urban area 1 for parents of type $a$. However, given Assumption 1, the net gains of socialization are higher in area 1 so that segregation forces dominate this integration force. Given that in all these cases the segregation forces dominate, at equilibrium, all trait $a$ individuals live in urban area 1 and decide to socialize their offspring. Both location and socialization choices are thus complements for the direct vertical socialization.

When $Q_t \in [\tilde{q}_2, 1]$, four urban configurations must be considered. First, if all type $a$ individuals live in urban area 1, i.e. $\tilde{q}_2 < q_t^a = Q_t$, the cultural substitution effect dominates peer effects leading parents to exert no socialization effort. In such a case, the bid-rent differential only depends on the oblique transmission effect. Second, both urban areas can be inhabited by trait $a$ individuals and the social composition in each area is such that individuals decide to exert a socialization effort, i.e. $\tilde{q}_1 < q_t^2 < q_1^a < \tilde{q}_2$. Segregation and integration forces are at play in (11). However, given Assumption 1, segregation forces dominate and lead (11) to be negative. Third, trait $a$ individuals reside in both urban areas. In urban area 1, they represent a high fraction of the population. The cultural substitution effect outweighing the peer effects, parents do not exert a socialization effort.
in this area. However, in urban area 2, the social composition is such that peer effects compensate the cultural substitution effect and parents exert an effort, i.e. \( \tilde{q}_1 < q^2_t < \tilde{q}_2 < q^1_t \). The bid-rent differential (11) depends on the oblique transmission effect and on the effect of socialization choice in urban area 2. This is the case where the integration force is the highest. In the Appendix, we show that there exists \( q^2_t \) and \( Q_t \) such that the segregation force and the integration force exactly compensate and make the bid-rent differential equal to 0. Nonetheless, this is an unstable equilibrium.

Fourth, type a individuals live in both urban areas and the social composition in each area is such that type a parents do not exert a socialization effort in both areas. In urban area 1, the reason is that the cultural substitution effect dominates, i.e. \( q^1_t > \tilde{q}_2 \), while in urban area 2 socialization gains are too low, i.e. \( q^2_t < \tilde{q}_1 \). As a whole, the equilibrium city is such that all trait a individuals reside in urban area 1 and exert no socialization effort. In this range of values of \( Q_t \), location decision and socialization efforts are substitutes for the direct vertical transmission of trait a.

As soon as \( Q_t \geq 1 \), both urban areas are inhabited by individuals with trait a. When \( Q_t \in [1, 1 + \tilde{q}_1] \) and \( Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2] \), the number of individuals with trait a is high and prevents from having the case \( q^2_t < \tilde{q}_1 < q^1_t < \tilde{q}_2 \). For both intervals, the three cases must be considered. First, the social composition in both urban areas can be such that peer effects are relatively high, while the cultural substitution effect is relatively low, i.e. \( \tilde{q}_1 < q^2_t < q^1_t < \tilde{q}_2 \). Hence, parents in both urban areas exert a socialization effort. Given Assumption 1, the segregation forces prevail leading the bid-rent differential to be negative. Second, the number of individuals of type a in urban area 1 is high and leads the cultural substitution effect to outweigh the peer effects. No socialization effort is exerted in urban area 1. However, in urban area 2, the social composition is such that the balance between peer effects and the cultural substitution effect is positive, i.e. \( \tilde{q}_1 < q^2_t < \tilde{q}_2 < q^1_t \). This is the case where the incentives to integrate are high. However, given Assumption 4, the integration force is outweighed by segregation forces so that no stable interior equilibrium exists. Finally, the social composition in both urban areas can be such that the cultural substitution effect dominates, i.e. \( \tilde{q}_2 < q^2_t < q^1_t \). In both urban areas, there is no incentives to exert a socialization effort. The bid-rent differential only depends on the oblique transmission effect and is thus negative. In all these cases, individuals with trait a outbid those with trait b in order to live in urban area 1.

When \( Q_t \in [1, 1 + \tilde{q}_1] \), the urban equilibrium is such that urban area 1 is only inhabited by type a individuals while urban area 2 is inhabited by a small fraction of trait a individuals. No socialization effort is exerted in both urban areas. The reason is that the cultural substitution effect is strong in urban area 1, and peers effects are low in urban area 2.
When \( Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2] \), the only difference with the previous urban equilibrium is that now the fraction of type \( a \) individuals living in urban area 2 is sufficient to ensure that, in this area, peers effects outweigh the cultural substitution effect.

When \( Q_t \in [1 + \tilde{q}_2, 2] \), the fraction of individuals with trait \( a \) is high so that, for any social composition in both urban areas, the cultural substitution effect outweighs peer effects, i.e. \( \tilde{q}_2 < q_t^2 < q_t^1 \). Parents do not exert any socialization effort. Hence, the bid-rent differential (11) is negative due to the oblique transmission effect. At equilibrium, all type \( b \) individuals reside in urban area 2.

Finally, social mobility prospects for the youth living in urban area 1 are better than for those living in urban area 2. Whatever the segregated equilibrium that emerges, the probability for a child whose parents have trait \( a \) to be socialized to this trait is always higher in urban area 1, i.e.

\[
P_{\text{bat}} = d_{1,a} + (1 - d_{1,a})q_t^1 > P_{\text{bat}} = d_{2,a} + (1 - d_{2,a})q_t^2.
\]

Furthermore, the prospect of upward social mobility for children whose parents have trait \( b \) is better in urban area 1, i.e.

\[
P_{\text{bat}} = q_t^1 > P_{\text{bat}} = q_t^2
\]

This result is consistent with the empirical findings of Chetty et al. (2014) on the spatial variation of intergenerational mobility\(^{11}\).

### 4 ‘Stationary Mobile’ versus ‘Dynamically Mobile’ Cities

We study the dynamic relationship between segregation and intergenerational mobility. Each urban equilibrium in Proposition 1 is characterized by particular dynamics of trait \( a \), that is, by a particular dynamics of the rate of education. Given (10), we can then offer

**Proposition 2** At any date \( t \), population dynamics can be characterized as follows

1. For any \( Q_t \in [0, \tilde{q}_1] \), \( Q_{t+1} = Q_t \).
2. For any \( Q_t \in [\tilde{q}_1, \tilde{q}_2] \), \( Q_{t+1} = Q_t + Q_t (1 - Q_t) \tau \).
3. For any \( Q_t \in [\tilde{q}_2, 1 + \tilde{q}_1] \), \( Q_{t+1} = Q_t \).
4. For any \( Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2] \), \( Q_{t+1} = Q_t + (Q_t - 1)(2 - Q_t) \tau \).
5. For any \( Q_t \in [1 + \tilde{q}_2, 2] \), \( Q_{t+1} = Q_t \).

This proposition highlights the interplay between the urban segregation and the population dynamics. Over the interval \([0, 2]\), the dynamics of education is either increasing or stationary. There are multiple history-dependent steady states.

\(^{11}\)Note that due to segregation, as long as \( Q_t \leq 1 \), then \( q_{2t} = 0 \) which implies that there is no prospect of upward social mobility in urban area 2.
When the fraction of educated agents is relatively reduced, i.e. \( Q_t \in [0, \tilde{q}_1] \), the population dynamics is stationary as no socialization choice is exerted in the city. This city is trapped in a low and stagnant level of education. The number of children of type \( a \) parents who experience downward social mobility, i.e. who acquires trait \( b \), exactly equals the number of children of type \( b \) parents who experience upward social mobility, i.e. who acquires trait \( a \). The city is called ‘stationary mobile’.

When the fraction of educated agents is higher, i.e. \( Q_t \in [\tilde{q}_1, \tilde{q}_2] \), the urban equilibrium is such that urban area 1 provides the incentives to exert a socialization effort. The education rate in urban area 1 increases overtime. This type of city is called ‘dynamically mobile’ because the number of children of type \( b \) parents who experience upward social mobility thanks to oblique transmission, i.e. who acquires trait \( b \), exceeds equals the number of children of type \( a \) parents who experience downward social mobility.

Nonetheless, the expansion of the the fraction of educated agents is stopped as soon as it reaches the threshold \( \tilde{q}_2 \). Indeed, when \( Q_t \in [\tilde{q}_2, 1] \), the urban segregation is such that all parents with trait \( a \) live in urban area 1 and do not find anymore profitable to exert a socialization choice due to a strong cultural substitution effect. The city is ‘stationary mobile’. As a whole any population starting with \( Q_0 \in [\tilde{q}_1, \tilde{q}_2] \) ends up at an intermediate level of the education rate \( Q_\infty \geq \tilde{q}_2 \).

When \( Q_t \in [1, 1 + \tilde{q}_1] \), the city is ‘stationary mobile’. Whatever the place of residence of educated individuals because they face no incentives to exert socialization choice. For larger fractions of the type \( a \) population, i.e. \( Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2] \) is relatively large, the city is ‘dynamically’ mobile as parents \( a \) living in urban area 2 socialize their offspring. Once again, the cultural trait \( a \) spreads out in the population and the rate of education increases. However, this expansion is halted as soon as the number of educated agents reaches the threshold \( 1 + \tilde{q}_2 \). When \( Q_t \in [1 + \tilde{q}_2, 2] \), the city is ‘stationary mobile’ because the cultural substitution effect is strong and both urban areas are deprived from any incentives to exert socialization effort.

We can also proceed to the following comparative static analysis

**Proposition 3**  
(i) An increase in \( \Delta w \), \( a \), \( \tau \) reduces the number of ‘stationary mobile’ cities, i.e. \( \tilde{q}_1 \) decreases and \( \tilde{q}_2 \) increases so that the set of values \( Q_t \) such that the dynamics is stationary decreases.

(ii) An increase in \( c \), \( \theta \) increases the number of ‘stationary mobile’ cities, i.e. \( \tilde{q}_1 \) increases and \( \tilde{q}_2 \) decreases so that the set of values \( Q_t \) such that the dynamics is stationary increases.

This Proposition stresses that the characteristics of the population dynamics depend on incentives to socialize. Precisely, when either returns to education, \( \Delta w \), the extra gain from education \( a \) or
the efficiency of the socialization effort, \( \tau \), increase then incentives to exert a socialization effort are strengthened. It turns out that the set of values of \( Q_t \) such that the population dynamics is stationary shrinks. Any population starting in \([\tilde{q}_1, \tilde{q}_2] \) or \([1 + \tilde{q}_1, 1 + \tilde{q}_2] \) will reach at the steady state a higher level of education. On the contrary, an increase in \( c \) or \( \theta \) lowers incentives for socialization effort. Any city will be likelier to reach a lower level of education at the steady state.

5 Is Social Segregation Optimal?

We now turn to the issue whether it is efficient to let people sort themselves into urban areas, or, put another way, whether it is desirable to implement specific urban policies (as quotas). The efficiency criterion we use is the long run rate of education.

**Proposition 4** For \( Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}] \cup [\max\{2\tilde{q}_2, 1 + \tilde{q}_1\}, 1 + \tilde{q}_2] \) segregation is efficient. For \( Q_t \in [\tilde{q}_2, 1 + \tilde{q}_1[ \) segregation is inefficient.

This proposition stresses that laissez-faire is not always optimal. When the rate of education is relatively low, \( Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}] \), and the urban equilibrium is segregated, the fraction of agents of type \( a \) in urban area 1 becomes sufficiently high so that peers effects create an incentive to transmit preferences for education in urban area 1. In this case, the city is ‘dynamically mobile’ and the long run rate of education increases. By contrast, if the city were mixed, the city would be ‘stationnary mobile’. Peer effects would be too low in both urban areas to create an incentive for parents of type \( a \) to transmit their preferences for education. Therefore, agents of type \( a \) would not transmit their trait in any area and the fraction of educated agents would not increase.

In the case where the rate of education is larger and such that \( Q_t \in [\max\{2\tilde{q}_2, 1 + \tilde{q}_1\}, 1 + \tilde{q}_2] \), segregation decreases the fraction of agents \( a \) in area 2 lowering the cultural substitution effect. Hence, urban area 2 provides incentives to transmit trait \( a \). By contrast, in a mixed city, the cultural substitution effect would turn out to be large in both urban areas so that parents of type \( a \) transmit their trait in no urban area and the fraction of educated agents does not increase.

A policy promoting integration may be optimal for intermediate values of \( Q_t \), i.e. \( Q_t \in [\tilde{q}_2, 1 + \tilde{q}_1[ \). When the city is segregated, the fraction of agents of type \( a \) is relatively high in area 1 and relatively low in area 2. Then, in area 2, peer effects are too low to create an incentive to transmit preferences for education. Besides, in area 1, since the fraction of agents \( a \) is high, the cultural substitution effect is too high so that these agents neither have an incentive to exert a socialization effort. Since, agents do not transmit preferences for education in both area, the rate of education is constant.
On the one hand, integration reduces the fraction of agents $a$ in area 1 so that peer effects may overcome the cultural substitution effect and agents $a$ have an incentive to transmit their trait in this area. On the other hand, integration increases the fraction of educated agents in area 2, so that peer effects now become sufficiently high to encourage agents of type $a$ to exert a socialization effort. Therefore, the fraction of educated agents increases and the long run rate of education is higher.\footnote{12}

A parallel can be made between these results and the empirical findings of Chetty et al. (2014) which stress that places with high segregation have low social mobility. Our model shows that the link between segregation and social mobility can also be viewed from a dynamic perspective. According to Proposition 4, we show that there are values of $Q_t$ such that segregation by depriving urban areas from incentives to transmit the trait $a$ lowers upward social mobility arising in the subsequent periods. However, we also show that there are other values of $Q_t$ such that segregation increases upward social mobility arising in the subsequent periods. In this case, segregation increases overtime oblique transmission which is the only way for children with type $b$-parents, to acquire preferences of type $a$.

There is room for efficiency improving public policies that would aim to affect the socio-economic composition of urban areas. The first obvious public intervention would be to enforce quotas in order to obtain more mixed urban areas. Moreover, our framework allows us to highlight other kinds of policies that would impact the parameters.

**Proposition 5** When parameters are such that

$$4(a + b)^2 < \tau(a + \Delta w) - 4c\theta,$$

if $Q_t \in [\tilde{q}_2, Q^*], Q^* \leq 1 + \tilde{q}_1$, integration and segregation are stable equilibria and integration is efficient.

In Section 3, we emphasize that for some values of $Q_t$, i.e. $Q_t \in [\tilde{q}_2, 1], Q_t \in [1, 1 + \tilde{q}_1]$ and $Q_t \in [1 + \tilde{q}_1, 1 + \tilde{q}_2]$, the social composition in urban areas can be such that the integration force is high. This is the case when the cultural substitution effect is such that agents make a socialization effort in area 2 and not in area 1. In such a case, indeed, the incentive to live in urban area 1 is reduced as parents $a$ can transmit their trait with a high probability in urban area 2 by exerting an
high socialization effort. If parameters are such that gains from socialization are high relative to gains from oblique transmission (see the above condition in Proposition 5), there is a social composition in urban areas such that the integration force outweighs segregation forces. Hence, a mixed/integrated equilibrium can emerge in addition to the segregated one. Moreover, as $q_2$ is not too low at the mixed equilibrium, parents of type $a$ transmit their trait in area 2 so that the fraction of educated agents increases and reaches a long run level of education higher than the one what reached if segregation holds.

This Proposition thus highlights that public policies can influence the magnitude of integration and segregation forces. Starting with parameters satisfying Assumption 4 leading the segregated city to be the unique equilibrium, there are various policies that could reverse inequality of Assumption 4 and give rise to an integrated and efficient equilibrium. One mean would be to increase returns to education, that is increase $\Delta w$ or reduce $c$. Another mean would be to influence the preferences parameters $a$ and $b$. These parameters can be the result of some social norm effects. Then our model allows to emphasize the role of a different kind of policy which aims to change social norm, that is group-based policies (Liu et al., 2014). An example is the charter school policy which aims to change behaviours bad for education by for instance implementing a very strict discipline or featuring a long-school day (see Angrist et al., 2013, Curto and Fryer, 2014).

6 Conclusion

We develop a model of neighbourhood formation and preference transmission that allows us to study the dynamic relationship between intergenerational mobility and the spatial socio-economic segregation. At each date, parents choose their place of residence and their effort in order to transmit their preferences for education. The trade-off faced by parents when deciding their socialization effort depends on the endogenous social composition of the urban areas via two effects, the cultural substitution effect and peer effects. Hence, depending on the social composition of the neighbourhood, parents may be willing to transmit their cultural trait or prefer to let their offspring acquire this trait by oblique transmission. Our model thus highlights two new forces driving the location decisions. The first one generated by oblique cultural transmission favours segregation. Indeed, due to imperfect empathy, parents prefer to live with similar individuals in order to get a higher chance that their offspring acquire their trait by oblique transmission. The second one comes from the substitutability between direct socialization and oblique transmission. Parents may prefer to live in
a mixed neighbourhood where oblique transmission is not effective in order to directly transmit their trait and augment the odds that their offspring acquire their trait. This is an integration force. First, we characterize at any date $t$ the segregated equilibrium that arises. Urban areas differ with respect to the education rates, and intergenerational mobility. Whatever his/her parents’ preferences, a child living in the urban area with the highest rate of education will more likely acquire the education trait. Second, we show that the joint dynamics of segregation and distribution of traits in the whole population exhibits multiple history-dependent steady states. Third, we show that laissez-faire will not always lead people to locate optimally. The inefficiency comes from an excessive concentration of people with preferences for education which generates a strong cultural substitution effect lowering incentives to exert a transmission effort.
References


Appendix

7.1 Assumptions 2 and 3

**Assumption 2** Let us denote by $P^a(q)$ the LHS of (7). It is a polynomial of order two, going from $[0, 1]$ into $\mathbb{R}$. It is concave with $P^a(1) = -\theta < 0$. Furthermore, $q^a_{\text{max}}$ which is such that $P^a(q^a_{\text{max}}) = 0$ is given by

$$q^a_{\text{max}} = 1 - \frac{\Delta w}{2c}.$$ 

Precisely, parameters $c$, $w_r$, $w_p$, $\tau$, $\theta$ are such that $P^a(q) = (1 - q)\tau (\Delta w - (1 - q)c + a) - \theta$ satisfies

(i) $P^a(0) = \tau (\Delta w - c + a) - \theta < 0$,
(ii) $q^a_{\text{max}} = 1 - \frac{\Delta w}{2c} > 0 \Leftrightarrow 2c > \Delta w$,
(iii) $P^a(q^a_{\text{max}}) > 0 \Leftrightarrow \left(\frac{\Delta w}{2c}\right)\tau \left(\frac{\Delta w}{2} + a\right) - \theta \geq 0$
(iv) $P^a\left(\frac{1}{2}\right) > 0 \Leftrightarrow \tau \left(\frac{\Delta w}{2} - \frac{c}{2} + a\right) > \theta$.

From items (i)-(iii) and $P^a(1) < 0$, one can deduce that $P^a(q)$ admits two positive roots $\tilde{q}_1$ and $\tilde{q}_2$ given by

$$\tilde{q}_1 = 1 - \frac{\tau(a + \Delta w) + \sqrt{D}}{2\tau c}, \quad \tilde{q}_2 = 1 - \frac{\tau(a + \Delta w) - \sqrt{D}}{2\tau c}$$

with

$$D = \tau \left(\tau(a + \Delta w)^2 - 4c\theta\right).$$

As $\tau < 1$, it is easy to see that $\tilde{q}_2 < 1$. It is also easy to show that $\tilde{q}_1 + \tilde{q}_2 < 1$. Finally, item (iv) implies that $\tilde{q}_1 < 1/2 < \tilde{q}_2$.

**Assumption 3** Let us denote by $P^b(q^j)$ the left-hand side of (9). The function $P^b : [0, 1] \rightarrow \mathbb{R}$ is concave with $P^b(0) = -\theta < 0$ and $P^b(1) = -\tau (\Delta w - b) - \theta$. Let us compute $q^b_{\text{max}}$ which is such that $P^b(q^b_{\text{max}}) = 0$, to find

$$q^b_{\text{max}} = \frac{1}{2} - \frac{(\Delta w - b)}{2c}.$$ 

We assume that parameters $\tau$, $\Delta w$, $c$, $b$ and $\theta$ are such that

$$P^b(q^b_{\text{max}}) < 0 \Leftrightarrow \frac{\tau}{c} \left(\frac{\Delta w - b}{2} - \frac{c}{2}\right)^2 - \theta < 0.$$
7.2 Proof of Proposition 1

For any \( Q_t \in [0, 2] \), and any \( q_t^1 \in [Q_t/2, Q_t] \), \( q_t^2 = Q_t - q_t^1 \in [0, Q_t/2] \), we study the bid-rent differential and characterize the implied urban equilibrium.

**Case 1** When \( Q_t \in [0, \tilde{q}_1] \), for any \( q_t^1 \in [Q_t/2, Q_t] \), and \( q_t^2 \in [0, Q_t/2] \), we have \( q_t^2 \leq q_t^1 \leq \tilde{q}_1 \).

We know from equation (8) that type \( a \) agents exert no effort in both urban areas. From (11), the rent differential then equals

\[
\bar{p}^{1} - \bar{p}^{a} = -(q_t^1 - q_t^2)[b + a] \leq 0 \text{ as } q_t^1 \geq q_t^2. \tag{12}
\]

**Result** For any \( Q_t \in [0, \tilde{q}_1] \), type \( a \) parents outbid type \( b \) parents and both \( [\rho_t^* = \bar{p}^{1}, q_t^{1*} = Q_t, d^{*,1,a} = 0, d^{*,2,a} = 0] \) and \( [\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*,1,a} = 0, d^{*,2,a} = 0] \) are urban equilibria. However, \( [\rho_t^* = 0, q_t^{1*} = Q_t/2, d^{*,1,a} = 0, d^{*,2,a} = 0] \) is unstable. Furthermore, type \( a \) parents exert no effort in both urban areas.

In all the cases that follows, suppose that \( 2\tilde{q}_1 < \tilde{q}_2 \).

**Case 2** When \( Q_t \in [\tilde{q}_1, \tilde{q}_2] \), we have various possible situations.

**Subcase 2.1** Suppose that \( 2\tilde{q}_1 < \tilde{q}_2 \) and \( Q_t \in [\tilde{q}_1, 2\tilde{q}_1] \).

*i/* Suppose that \( q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_1] \), \( q_t^2 \in [Q_t - \tilde{q}_1, \frac{Q_t}{2}] \), which implies \( q_t^2 \leq q_t^1 \leq \tilde{q}_1 \). Type \( a \) agents exert no effort in both urban areas. The rent differential is (12).

**ii/* Suppose that \( q_t^1 \in [\tilde{q}_1, Q_t] \), \( q_t^2 \in [0, Q_t - \tilde{q}_1] \), then \( q_t^2 \leq \tilde{q}_1 \leq q_t^1 < \tilde{q}_2 \). Type \( a \) agents exert effort of socialization \( \tau \) in area 1 and no effort in urban area 2. From (11), the rent differential equals

\[
\bar{p}^{1} - \bar{p}^{a} = -(q_t^1 - q_t^2)[b + a] - \tau (1 - q_t^1) \left( \Delta V_{a}^{1} \right) + \theta. \tag{13}
\]

As parents with trait \( a \) have an incentive to exert \( \tau \) we know that \( -\tau (1 - q_t^1) \left( \Delta V_{a}^{1} \right) + \theta < 0 \) leading to

\[
\bar{p}^{1} - \bar{p}^{a} < 0.
\]

\(^{13}\)The proof for the case with \( 2\tilde{q}_1 > \tilde{q}_2 \) is very similar so that we omit the details to limit the number of cases.
Type \( a \) parents outbid type \( b \) parents.

**Subcase 2.2** Suppose that \( Q_t \in [2 \bar{q}_1, \bar{q}_2] \).

\( i/ \) When \( q^1 \in [\frac{Q_t}{2}, Q_t - \bar{q}_1], q^2 \in [\bar{q}_1, \frac{Q_t}{2}] \), then \( \bar{q}_1 \leq q^2_t \leq q^1_t < \bar{q}_2 \). Type \( a \) agents exert effort in both urban areas. From (11), (4) and (6), the rent differential then equals

\[
\bar{p}_t^{b,1} - \bar{p}_t^{a,1} = (q^1_t - q^2_t) \left[-b - a - \tau (c - \Delta w - a + c (1 - Q_t))\right].
\] (14)

This rent differential increases with \( Q_t \). For \( Q_t = 2 \), we have

\[
\bar{p}_t^{b,1} - \bar{p}_t^{a,1} = (q^1_t - q^2_t) \left[-b + \tau \Delta w - (1 - \tau)a\right],
\]

which is negative by Assumption 1. Therefore, the rent differential (14) is negative for any \( Q_t \in [0, 2] \) and \( q^1_t \geq q^2_t \). Type \( a \) individuals outbid type \( b \) individuals.

\( ii/ \) When \( q^1_t \in [Q_t - \bar{q}_1, Q_t], q^2_t \in [0, \bar{q}_1] \), then \( q^2_t \leq \bar{q}_1 < q^1_t < \bar{q}_2 \). Type \( a \) agents exert effort in urban area 1 and no effort in urban area 2 so that the rent differential is given by (13) and type \( a \) individuals outbid type \( b \) individuals.

To sum up, when \( Q_t \in [\bar{q}_1, \bar{q}_2] \),

If \( Q_t \in [\bar{q}_1, 2\bar{q}_1] \), the rent differential is

\[
\begin{align*}
(12) & \text{ when } q^1_t \in [\frac{Q_t}{2}, \bar{q}_1], q^2_t \in [Q_t - \bar{q}_1, \frac{Q_t}{2}], \\
(13) & \text{ when } q^1_t \in [\bar{q}_1, Q_t], q^2_t \in [0, Q_t - \bar{q}_1].
\end{align*}
\]

If \( Q_t \in [2\bar{q}_1, \bar{q}_2] \), the rent differential is

\[
\begin{align*}
(14) & \text{ when } q^1_t \in [\frac{Q_t}{2}, Q_t - \bar{q}_1], q^2_t \in [\bar{q}_1, \frac{Q_t}{2}], \\
(13) & \text{ when } q^1_t \in [Q_t - \bar{q}_1, Q_t], q^2_t \in [0, \bar{q}_1].
\end{align*}
\]

**Result** For any \( Q_t \in [\bar{q}_1, \bar{q}_2] \),

- when \( Q_t \in [\bar{q}_1, 2\bar{q}_1] \), \( [\rho^*_t = 0, q^1_t^* = Q_t/2, d^{*1,a} = 0, d^{*2,a} = 0] \) and \( [\rho^*_t = \bar{p}_t^{b,1}, q^1_t^* = Q_t, d^{*1,a} = \tau, d^{*2,a} = 0] \) are urban equilibria. As the first one is unstable, the second one is the unique urban equilibrium;

- when \( Q_t \in [2\bar{q}_1, \bar{q}_2] \), \( [\rho^*_t = 0, q^1_t^* = Q_t/2, d^{*1,a} = \tau, d^{*2,a} = \tau] \) and \( [\rho^*_t = \bar{p}_t^{b,1}, q^1_t^* = Q_t, d^{*1,a} = \tau, d^{*2,a} = 0] \) are urban equilibria. As the first one is unstable, the second one is the unique urban equilibrium.
Case 3 When $Q_t \in [\tilde{q}_2, 1]$.

Subcase 3.1 $Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2]$.

i/ $q^1_t \in [\frac{Q_t}{2}, Q_t - \tilde{q}_1]$, $q^2_t \in [\tilde{q}_1, \frac{Q_t}{2}]$ then $\tilde{q}_1 \leq q^2_t \leq q^1_t \leq \tilde{q}_2$ type $a$ agents exert $\tau$ in both urban areas. The rent differential is given by (14).

ii/ $q^1_t \in [Q_t - \tilde{q}_1, \tilde{q}_2]$, $q^2_t \in [Q_t - \tilde{q}_2, \tilde{q}_1]$ then $q^2_t \leq \tilde{q}_1 \leq q^1_t \leq \tilde{q}_2$ type $a$ agents exert effort in urban area 1 and no effort in urban area 2. The rent differential is given by (13).

iii/ $q^1_t \in [\tilde{q}_2, Q_t]$, $q^2_t \in [0, Q_t - \tilde{q}_2]$ then $q^2_t \leq \tilde{q}_1 < \tilde{q}_2 \leq q^1_t$ type $a$ agents exert no effort in both urban areas. The rent differential is given by (12).

Subcase 3.2 $Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1]$

i/ $q^1_t \in [\frac{Q_t}{2}, \tilde{q}_2]$, $q^2_t \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}]$ then $\tilde{q}_1 \leq q^2_t \leq q^1_t \leq \tilde{q}_2$ type $a$ agents exert $\tau$ in both urban areas. The rent differential is given by (14).

ii/ $q^1_t \in [\tilde{q}_2, Q_t - \tilde{q}_1]$, $q^2_t \in [\tilde{q}_1, Q_t - \tilde{q}_2]$ then $\tilde{q}_1 \leq q^2_t \leq \tilde{q}_2 \leq q^1_t$ type $a$ agents exert no effort in urban area 1 and exert effort $\tau$ in urban area 2. From (11), the rent differential equals

$$\overline{p}^b_{t,1} - \overline{p}^a_{t,1} = -(q^1 - q^2)[b + a] + \tau (1 - q^2) (\Delta V^2_a) - \theta. \tag{15}$$

So that

$$\overline{p}^b_{t,1} - \overline{p}^a_{t,1} \leq 0 \iff (Q_t - 2q^2_t)[b + a] \geq \tau (1 - q^2) (\Delta V^2_a) - \theta$$

Given equation (8), as $q^2_t \in [\tilde{q}_1', \tilde{q}_2']$, we know that $\tau (1 - q^2) (\Delta V^2_a) - \theta \geq 0$.

As $Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1]$ and that $2q^2 > 1$, we deduce that $2q^1 < Q_t < 2q^2$. Hence, for any $Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1]$, $(Q_t - 2q^1)[b + a] > 0$ and $(Q_t - 2q^2)[b + a] < 0$. We deduce that there exists a unique solution $q^{**}$ such that $(Q_t - 2q^{**})[b + a] = \tau (1 - q^{**}) (\Delta V^2_a) - \theta$. This is an unstable equilibrium. To show this consider a small perturbation $\epsilon > 0$. One has $-(Q_t - 2(q^{**} - \epsilon))[b + a] + (\tau (1 - (q^{**} - \epsilon)) (\Delta V^2_a) - \theta) < 0$. It means that a small perturbation which consists to increase by an infinitesimal amount the number of agents $a$ in area 1, creates an incentive for these agents, to migrate into area 1 so that this equilibrium is unstable.

iii/ $q^1_t \in [Q_t - \tilde{q}_1, Q_t]$, $q^2_t \in [0, \tilde{q}_1]$ then $q^2_t \leq \tilde{q}_1 \leq \tilde{q}_2 \leq q^1_t$ type $a$ agents exert no effort in both urban
areas. The rent differential is given by (12).

To sum up, when \( Q_t \in [\tilde{q}_2, 1] \),

if \( Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2] \), the rent differential is

\[
\begin{align*}
(14) & \text{ when } q_1^t \in \left[ \frac{Q_t}{2}, Q_t - \tilde{q}_1 \right], q_2^t \in \left[ \tilde{q}_1, \frac{Q_t}{2} \right], \\
(13) & \text{ when } q_1^t \in \left[ Q_t - \tilde{q}_1, \tilde{q}_2 \right], q_2^t \in \left[ Q_t - \tilde{q}_2, \tilde{q}_1 \right], \\
(12) & \text{ when } q_1^t \in \left[ \tilde{q}_2, Q_t \right], q_2^t \in \left[ 0, Q_t - \tilde{q}_2 \right],
\end{align*}
\]

if \( Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1] \), the rent differential is

\[
\begin{align*}
(14) & \text{ when } q_1^t \in \left[ \frac{Q_t}{2}, \tilde{q}_2 \right], q_2^t \in \left[ Q_t - \tilde{q}_2, \frac{Q_t}{2} \right], \\
(15) & \text{ when } q_1^t \in \left[ \tilde{q}_2, Q_t - \tilde{q}_1 \right], q_2^t \in \left[ \tilde{q}_1, Q_t - \tilde{q}_2 \right], \\
(12) & \text{ when } q_1^t \in \left[ Q_t - \tilde{q}_1, Q_t \right], q_2^t \in \left[ 0, \tilde{q}_1 \right].
\end{align*}
\]

**Results** For any \( Q_t \in [\tilde{q}_2, 1] \),

- when \( Q_t \in [\tilde{q}_2, \tilde{q}_1 + \tilde{q}_2] \), \([\rho_t^* = \frac{\rho_t^1}{\tilde{p}_t^1}, q_1^t = q_1^t, d^{t,1} = 0, d^{t,2} = 0]\) and \([\rho_t^* = 0, q_1^t = \frac{Q_t}{2}, d^{t,1} = \tau, d^{t,2} = \tau]\) are the urban equilibria. As the first one is unstable, the second one is the unique equilibrium which arises;

- when \( Q_t \in [\tilde{q}_1 + \tilde{q}_2, 1] \), \([\rho_t^* = \frac{\rho_t^1}{\tilde{p}_t^1}, q_1^t = q_1^t, d^{t,1} = 0, d^{t,2} = 0]\) and \([\rho_t^* = 0, q_1^t = \frac{Q_t}{2}, d^{t,1} = \tau, d^{t,2} = \tau]\) are urban equilibria. Only the first one arises since the second one is unstable.

In what follows we assume, \( 2\tilde{q}_2 > 1 + \tilde{q}_1 \).

**Case 4** Suppose \( Q_t \in [1, 1 + \tilde{q}_1] \).

**Subcase 4.1** \( Q_t \in [1, 2\tilde{q}_2] \).

i/ \( q_1^t \in \left[ \frac{Q_t}{2}, \tilde{q}_2 \right], q_2^t \in \left[ Q_t - \tilde{q}_2, \frac{Q_t}{2} \right], \) then \( \tilde{q}_1 \leq q_2^t \leq q_1^t \leq \tilde{q}_2 \), type \( a \) agents exert effort in both urban areas. The rent differential is given by (14).

ii/ \( q_1^t \in \left[ \tilde{q}_2, Q_t - \tilde{q}_1 \right], q_2^t \in \left[ \tilde{q}_1, Q_t - \tilde{q}_2 \right], \) then \( \tilde{q}_1 \leq q_2^t \leq \tilde{q}_2 \leq q_1^t \). Type \( a \) agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (15).

iii/ \( q_1^t \in \left[ Q_t - \tilde{q}_1, 1 \right], q_2^t \in \left[ Q_t - 1, \tilde{q}_1 \right], \) then \( q_2^t \leq \tilde{q}_1 \leq \tilde{q}_2 \leq q_1^t \). Type \( a \) agents exert no effort in both

\[\text{As well, the proof for the case } 2\tilde{q}_2 > 1 + \tilde{q}_1 \text{ is very similar. We omit the details to simplify the presentation.}\]
urban areas. The rent differential is given by (12).

**Subcase 4.2** \( Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1] \).

\[ i/ \quad q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_2], \quad q_t^2 \in [\tilde{q}_2, \frac{Q_t}{2}], \text{ then } \tilde{q}_2 \leq q_t^2 \leq q_t^1. \]

Type \( a \) agents exert no effort in both urban areas. The rent differential is given by (12).

\[ ii/ \quad q_t^1 \in [Q_t - \tilde{q}_2, Q_t - \tilde{q}_1], \quad q_t^2 \in [\tilde{q}_1, \tilde{q}_2], \text{ then } \tilde{q}_1 \leq q_t^2 \leq \tilde{q}_2 \leq q_t^1. \]

Type \( a \) agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (15). When \( Q_t > 2\tilde{q}_2 \) a sufficient condition in order to have the rent differential (15) negative is that the slope of the line given by \((Q_t - 2\tilde{q}_2)[b + a]\) is lower than the slope of the curve which is given by \(\tau \left( 1 - q_t^2 \right) (\Delta V_a^2) - \theta\) in \(q_2 = \tilde{q}_2\), that is

\[ 2(b + a) > 2\tau c(1 - \tilde{q}_2^2) - \tau(\Delta w + a). \]

Let replace \( \tilde{q}_2^2 \) by its value. After some computations one gets

\[ 4(a + b)^2 > \tau(a + \Delta w) - 4c\theta. \]

This is true given Assumption 4.

\[ iii/ \quad q_t^1 \in [Q_t - \tilde{q}_1, 1], \quad q_t^2 \in [Q_t - 1, \tilde{q}_1], \text{ then } q_t^2 \leq q_t^1 \leq \tilde{q}_2 \leq q_t^1. \]

Type \( a \) agents exert no effort in both urban areas. The rent differential is given by (12).

To sum up when \( Q_t \in [1, 1 + \tilde{q}_1] \),

\[
\begin{align*}
\text{if } \quad Q_t \in [1, 2\tilde{q}_2], \quad \text{the rent differential is} & \quad \left\{ \begin{array}{l}
(14) \quad \text{when } q_t^1 \in [\frac{Q_t}{2}, \tilde{q}_2], \quad q_t^2 \in [Q_t - \tilde{q}_2, \frac{Q_t}{2}],
(15) \quad \text{when } q_t^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1], \quad q_t^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2],
(12) \quad \text{when } q_t^1 \in [Q_t - \tilde{q}_1, 1], \quad q_t^2 \in [Q_t - 1, \tilde{q}_1],
\end{array} \right.
\end{align*}
\]

\[
\begin{align*}
\text{if } \quad Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1], \quad \text{the rent differential is} & \quad \left\{ \begin{array}{l}
(12) \quad \text{when } q_t^1 \in [\frac{Q_t}{2}, Q_t - \tilde{q}_2], \quad q_t^2 \in [\tilde{q}_2, \frac{Q_t}{2}],
(15) \quad \text{when } q_t^1 \in [Q_t - \tilde{q}_2, Q_t - \tilde{q}_1], \quad q_t^2 \in [\tilde{q}_1, \tilde{q}_2],
(12) \quad \text{when } q_t^1 \in [Q_t - \tilde{q}_1, 1], \quad q_t^2 \in [Q_t - 1, \tilde{q}_1],
\end{array} \right.
\end{align*}
\]

**Result** For any \( Q_t \in [1, 1 + \tilde{q}_1] \),

- when \( Q_t \in [1, 2\tilde{q}_2], [\rho_t^1 = 0, q_t^{1*} = Q_t/2, d^{t, a} = \tau, d^{t, 2a} = \tau] \) is an urban equilibrium. It is unstable.
Hence the equilibrium that arises is \( \rho_t^* = \bar{p}_t^{a,1}, q_t^{1*} = 1, d^{s1,a} = 0, d^{s2,a} = 0 \),
- when \( Q_t \in [\bar{q}_2, 1 + \bar{q}_1] \), \( \rho_t^* = 0, q_t^{1*} = Q_t/2, d^{s1,a} = 0, d^{s2,a} = 0 \) is an urban equilibrium. It is unstable. The only equilibrium that arises is \( \rho_t^* = \bar{p}_t^{a,1}, q_t^{1*} = 1, d^{s1,a} = 0, d^{s2,a} = 0 \).

**Case 5** Suppose that \( Q_t \in [1 + \bar{q}_1, 1 + \bar{q}_2] \).

i/ \( q_t^1 \in [Q_t/2, Q_t - \bar{q}_2], q_t^2 \in [\bar{q}_2, Q_t/2], \) then \( \bar{q}_2 \leq q_t^2 \leq q_t^1 \). Type \( a \) agents exert no effort in both urban areas. The rent differential is given by (12).

ii/ \( q_t^1 \in [Q_t - \bar{q}_2, 1], q_t^2 \in [Q_t - 1, \bar{q}_2] \) then, \( \bar{q}_1 \leq q_t^2 \leq \bar{q}_2 \leq q_t^1 \). Type \( a \) agents exert no effort in urban area 1 and effort in urban area 2. The rent differential is given by (15).

To sum up when \( Q_t \in [1 + \bar{q}_1, 1 + \bar{q}_2] \),

the rent differential is

\[
\begin{align*}
(12) & \text{ when } q_t^1 \in [Q_t/2, Q_t - \bar{q}_2], q_t^2 \in [\bar{q}_2, Q_t/2], \\
(15) & \text{ when } q_t^1 \in [Q_t - \bar{q}_2, 1], q_t^2 \in [Q_t - 1, \bar{q}_2].
\end{align*}
\]

**Result** For any \( Q_t \in [1 + \bar{q}_1, 1 + \bar{q}_2] \),

\( \rho_t^* = 0, q_t^{1*} = Q_t/2, d^{s1,a} = 0, d^{s2,a} = 0 \) is an urban equilibrium. It is unstable. The equilibrium such that \( \rho_t^* = \bar{p}_t^{a,1}, q_t^{1*} = 1, d^{s1,a} = 0, d^{s2,a} = \tau \) is the only equilibrium that arises.

**Case 6** When \( Q_t \in [1 + \bar{q}_2, 2] \), we have, for \( q_t^1 \in [Q_t/2, 1], q_t^2 \in [Q_t - 1, Q_t/2], \) which implies \( \bar{q}_2 \leq q_t^2 \leq q_t^1 \). Type \( a \) agents exert no effort in both urban areas. The rent differential is given by (12).

**Result** \( \rho_t^* = 0, q_t^{1*} = Q_t/2, d^{s1,a} = 0, d^{s2,a} = 0 \) is an urban equilibrium. It is unstable. The unique equilibrium that arises is \( \rho_t^* = \bar{p}_t^{a,1}, q_t^{1*} = 1, d^{s1,a} = 0, d^{s2,a} = 0 \).

### 7.3 Proof of Proposition 4

Suppose that \( Q_t \in [\bar{q}_2, 1] \) then \( Q_{t+1} = Q_t \) so that \( Q = Q_t \) is the long run rate of education. At time \( t \), there is an integration policy which consists to transfer a fraction \( \Delta_t \) of agents of type \( a \) from area 1 into area 2 and which is such that \( \bar{q}_1 < q_t \Delta_t < \bar{q}_2 \). Given equation (8), type \( a \) agents now exert a socialization in area 1 and either \( \tau \) or 0 in area 2 depending on \( Q_t \) and \( \Delta_t \). In both cases, the dynamics is such that \( Q_{t+1} > Q_t \). The long run equilibrium is \( \bar{Q} = 1 > Q_t \).
Suppose that $Q_t \in [1, 1 + \tilde{q}_1]$ then $Q_{t+1} = Q_t$ so that $\bar{Q} = Q_t$ is the long run rate of education. At time $t$, there is an integration policy which consists to transfer a fraction $\Delta_t$ of agents of type $a$ from area 1 into area 2 and which is such that $\tilde{q}_1 < q_{2t} + \Delta_t < \tilde{q}_2$. Given equation (8), type $a$ agents now exert a socialization in area 2 and and either $\tau$ or 0 in area 2 depending $Q_t$ and $\Delta_t$. The dynamics is such that $Q_{t+1} > Q_t$. The long run equilibrium is $\bar{Q} = 1 + \tilde{q}_1 > Q_t$.

Suppose that $Q_t \in [\tilde{q}_1, \min\{2\tilde{q}_1, \tilde{q}_2\}]$, then $Q_{t+1} = Q_t + (1 - Q_t)Q_t\tau$ and given Proposition 2, $\bar{Q} = \tilde{q}_2$. Suppose, for instance, that at time $t$ there is an integration policy such that $q_{1t} = \frac{Q_t}{2}$, then $q_{1t} = q_{2t} < \tilde{q}_1$. Given Equation (8), type $a$ agents do not exert a socialization in area 1 nor in area 2. The dynamics is given by $Q_{t+1} = Q_t$ so that $\bar{Q} = \frac{Q_t}{2} < \tilde{q}_2$.

Suppose that $Q_t \in [\tilde{q}_1, \max\{2\tilde{q}_2, 1 + \tilde{q}_1\}]$, the dynamics is given by $Q_{t+1} = Q_t$ so that the long run equilibrium is $\bar{Q} = Q_t$. For any integration policy $q_{1t} < \tilde{q}_1$, the dynamics does not change. The long run equilibrium is $\bar{Q} = Q_t$. The proof for other cases is very similar so that we skip the details.

### 7.4 Proof of Proposition 5

Suppose that

$$4(a + b)^2 < \tau(a + \Delta w) - 4c\theta.$$ 

Consider $Q_t \in [2\tilde{q}_2, 1 + \tilde{q}_1]$. If $q_{1t}^1 \in [\tilde{q}_2, Q_t - \tilde{q}_1]$, $q_{1t}^2 \in [\tilde{q}_1, Q_t - \tilde{q}_2]$, then $\tilde{q}_1 \leq q_{1t}^2 \leq q_{1t}^1 \leq q_{1t}^1$ and the rent differential is given by (15).

Let us define the function $f$ and $g$ going from $[0, 1]$ into $\mathbb{R}$ which are such that 

$$f(q_2; Q_t) = (Q_t - 2q_{1t}^2)[b + a],$$

$$g(q_2) = \tau (1 - q_{1t}^2) (\Delta V_a^2) - \theta.$$
At $Q_t = 2\tilde{q}_2$, the equation $f(q_2; 2\tilde{q}_2) - g(q_2) = 0$ admits two solutions $q_2 = \tilde{q}_2$ and some $q_2 < \tilde{q}_2$ (given the proof of subcase 3.2). This is equivalent to say that the lines whose equation is given by $f(q_2; 2\tilde{q}_2)$, say $L_1$, is a chord of the function $g$. By the mean value theorem, one deduces that for some $q_2^* < \tilde{q}_2$ it exists a line $L_2$, parallel to $L_1$, which is tangent to the curve described by $g$.

We deduce that the equation of $L_2$ is given $f(q_2; Q^*)$, with $Q^* > 2\tilde{q}_2$ since $g$ is concave. Besides, due to the concavity of $g$, all the lines between $L_1$ and $L_2$ are chords of $g$. The set of these lines is described by the function $f(q_2; Q_t)$ when $Q_t$ varies from $2\tilde{q}_2$ to $Q^*$. This is equivalent to say that for $Q_t \in [2\tilde{q}_2, Q^*]$, the equation $f(q_2; 2\tilde{q}_2) - g(q_2) = 0$ admits two solutions inferior to $\tilde{q}_2$.

The two solutions correspond to interior equilibria: one stable and one unstable$^{15}$.

For $Q_t \in [2\tilde{q}_2, \min\{Q^*, 1+\tilde{q}_1\}]$, suppose that the interior stable equilibrium is selected. Given Proposition 4, the population dynamics is such that $Q_{t+1} > Q_t$ so that the long run rate of education is $\bar{Q} = \min\{Q^*, 1+\tilde{q}_1\}$. Now suppose that the segregated equilibrium is selected. Given Proposition 4, the dynamics is given by $Q_{t+1} = Q_t$ so that the long run rate of education is $\bar{Q} = Q_t < \min\{Q^*, 1+\tilde{q}_1\}$.

$^{15}$Arguments for stability are similar than the ones used in subcase 3.2 so that we skip the details.